Modelling and Analysis of Transformer under Non-Sinusoidal Current Excitation

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A Thesis submitted in partial fulfillment of the requirements for the degree of

Bachelor of Technology in "Electrical Engineering"

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CERTIFICATE

This is to certify that the thesis entitled "**Modelling and Analysis of Transformer under Non-Sinusoidal Current Excitation**", submitted by **Soumya Ranjan Mohanty (108EE086)** in partial fulfilment of the requirements for the award of **Bachelor of Technology** in **Electrical Engineering** during session 2011-2012 at National Institute of Technology, Rourkela. A bonafide record of research work carried out by them under my supervision and guidance.

The candidates have fulfilled all the prescribed requirements.

The Thesis which is based on candidates' own work, have not submitted elsewhere for a degree/diploma.

In my opinion, the thesis is of standard required for the award of a bachelor of technology degree in Electrical Engineering.

Place: Rourkela

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B.Tech(Electrical engg)

ABSTARCT

The transformers are an integral part of the power system. Due to the non-linear loads, the transformers are much affected by the distorted currents and supply voltages which largely reduce its efficiency due to overheating. Thus analysing this problem and reducing the losses of transformer has become a major area of research in today's scenario. This report includes the effects of non-sinusoidal supply voltage on the transformer excitation current and the core losses which includes eddy current and hysteresis losses. The time domain waveform of excitation current and flux are observed. The equivalent circuit model of the transformer under no load was taken to observe the waveforms. The fast Fourier transform of the excitation current is performed to analyse the harmonics and the total harmonic distortion is shown for various harmonic components. The losses are calculated assuming the flux density to be sinusoidal. The modelling of the transformer is performed for linear core i.e without saturation and also for non-linear core i.e with saturation effects. The dynamic modelling equations of the transformer are derived and are solved to obtain the flux, magnetizing current and curve. The whole system is studied in the MATLAB-Simulink environment and the corresponding results are obtained with detailed analysis.

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NOMENCLATURE

P_h = Hysteresis loss in the Iron Core

σ_h = Density of the Material

P_e = Eddy current loss in the Core

σ_e = Density of the Material

f = Power System Frequency

T = Time period of one cycle

I_s = Supply Current

I_1 = Fundamental component of Current

I_{rms} = Rms value of the Supply Current

$B(t)$ = Magnetic flux Density

$H(t)$ = Magnetic Field Intensity

φ_1 = Magnetic Flux in Primary Winding

φ_2' = Magnetic Flux density in Secondary Winding

φ_m^{sat} = Mutual Saturated flux linking the Core

r_1 = Primary Winding Resistance

r_2 = Secondary Winding Resistance

x_{l1} = Primary Winding Reactance

x_{l2} = Secondary Winding Reactance

x_m = Mutual Reactance

Chapter **1**

Introduction

1.1 Motivation:

Transformers are found in all parts of the power system, between all voltage levels, and exist in different sizes, types and connections.\Due to nonlinear loads, power frequency harmonics exist mainly in distribution systems. The non-linear loads that produce harmonics on the power system are static converters, rectifiers, arc furnaces, electronic phase control, cycloconvertors, switch mode power supplies, pulse width modulated drives, etc. A large part of loads, which draw nonsinusoidal current, is fed through power-electronics converters. The current harmonics cause voltage harmonics, hence resulting in nonsinusoidal supply voltage in the system. Harmonic voltage distortion may cause additional losses and heating in laminated transformer cores due to the distorted flux waveform [1].

Correct prediction of the losses under a distorted flux waveform is therefore an important prerequisite for the transformer design, particularly when stringent efficiency standards are required. Nonlinearity in the core dynamics significantly influences the behaviour of the transformer in the analysis of electro-magnetic transients.

The 3rd harmonic component in the supply voltage has the highest effect on the current harmonics and distortion. The effect of higher voltage harmonics gradually diminishes with frequency. The magnitudes of the individual current harmonics follow the same decreasing or increasing pattern of changes as does current distortion variation. The corresponding variation in harmonic current phase angles is negligible [2].

The primary effect of harmonic currents on transformers is the additional heat generated by the losses caused by the harmonic contents generated by the nonlinear loads.
There are three effects that result in increased transformer heating when the load current includes harmonic components.

1. Rms current: If the transformer is sized only for the kVA requirements of the load, harmonic currents may result in the transformer rms current being higher than its capacity.

2. Eddy-current losses: These are induced currents in a transformer caused by the magnetic fluxes.

3. Core losses: The increase in nonlinear core losses in the presence of harmonics will be dependent under the effect of the harmonics on the applied voltage and design of the transformer core [3].

According to Strategies for development and diffusion of Energy Efficient Distribution Transformers (SEEDT), the losses caused by harmonics and reactive power in European Union (EU) distribution transformers are estimated at about 5000 GWh/year. However, total losses of distribution transformers in EU (European Union) reach to 38000 GWh/year. Therefore harmonic analysis with calculations plays an important role in transformers to reduce harmonics effect [4].

1.2 Literature Review:

In power transformers the main consequence of harmonic currents is an increase in losses, mainly in windings, because of the deformation of the leakage fields. Higher losses mean that more heat is generated in the transformer so that the operating temperature increases, leading to deterioration of the insulation and a potential reduction in lifetime.

As a result, it is necessary to reduce the maximum power load on the transformer, a practice referred to as de-rating, or to take extra care in the design of the transformer to reduce these losses.

To estimate the de-rating of the transformer, the load's K-factor may be used. This factor is calculated according to the harmonic spectrum of the load current and is an indication of the additional eddy current load losses. It reflects the excess losses experienced in a traditional wire wound transformer.

Modern transformers use alternative winding designs such as foil windings or mixed wire/foil windings. For these transformers, the standardised K-factor – derived for the load current - does not reflect the additional load losses and the actual increase in losses proves to be very dependent on the construction method. It is therefore necessary to minimise the additional losses at the design stage of the transformer for the given load data using field simulation methods or measuring techniques [5].

Direct switching of the transformer to a network gives rise to a transition phenomenon with associated current surge. The flux starting from remnant flux changes so as to make its derivative vary in accordance with imposed network voltage which results in drawing heavy exciting current. This large transient current called magnetic inrush current also causes flux saturation.

1.3 Thesis Objectives:

The following objectives are hopefully to be achieved at the end of the project:

a) Transformer operation with sinusoidal and non-sinusoidal excitation and observing the transformer excitation current at no load.

b) Analysing the excitation current with the Fourier analysis and calculating each harmonic contribution. Also calculating the eddy current and hysteresis losses for operating/knee point.

c) Modelling of transformer from its equivalent circuit model based on the electromagnetic field of the transformer and incorporating magnetic saturation into the transformer model by using the saturation curve of transformer.

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1.4 Thesis Organisation:

The thesis is organised into five chapters including the chapter of introduction. Each chapter is different from the other and is described along with the necessary theory required to comprehend it.

Chapter 2 deals with the introduction of the transformer, basic laws governing the transformer operation, basic equations of transformer. Then it focusses on the losses occurring in the transformer and their description with equations for calculation of the losses. Derating of the transformer is then detailed in this section along with the introduction of K-factor. Then a detailed description is given on the Fourier series, for the analysis of the power system harmonics, alongside this the major parameter to measure the harmonic distortion , Total Harmonic Distortion (THD) is provided.

Chapter 3 deals with the equivalent circuit model of the transformer, the formation of exact equivalent circuit model from the original model with some assumptions. Then it deals with the saturation effects in the transformer, the main reasons behind the saturation in the core, effect of saturation on the excitation current waveform with sinusoidal supply and non-sinusoidal supply voltage. The Simulink models are provided for these effects. Then the detailed modelling of the hysteresis loss is given. Finally the modelling of transformer with core and without core saturation is provided with detailed mathematical modelling and their Simulink model.

Chapter 4 presents the experimental results. The open circuit and the short circuit test results are provided of the single phase transformer. The simulation results of the transformer modelling are also presented along with the harmonic analysis of the excitation and the mutual flux waveform.

Chapter 5 concludes the work performed with some experimental observations. The future work that can be done is mentioned.

CHAPTER **2**

Transformers De-rating and Power system Harmonics Analysis

2.1 Introduction:

A transformer is a device that transfers electrical energy from one circuit to another through inductively coupled conductors. A varying current in the primary winding creates a varying magnetic flux in the transformer's core and thus a varying magnetic field through the secondary winding. This varying magnetic field induces a varying electromotive force (EMF) or voltage in the secondary winding. This effect is called inductive coupling.

If the secondary winding is connected to a load, then the EMF induced in the secondary winding causes a current to flow through it which creates a MMF which opposes its cause. This reduces the magnetic flux of the core, thus the supply provides extra current to neutralise the opposing MMF. Thus the net MMF in the magnetic core is the difference of the MMF of the primary current and that produced by the attachment of load. This MMF corresponds to the no load current magnitude and is constant in the core while in operation.

The transformer action is dependent on Faraday's laws of induction which states:

$$Vs = Ns * (\partial\phi / \partial t) \tag{2.1.1}$$

$$Vp = Np * (\partial\phi / \partial t) \tag{2.1.2}$$

Where Ns and Np is the number of turns in the secondary and primary , ϕ is the magnetic flux through one turn of the coil, Vs and Vp is the instantaneous voltage of the secondary and primary respectively.

Taking the ratio of the two equations for Vs and Vp gives the equation for stepping up or stepping down the voltage of the transformer:

$$Vs / Vp = Ns / Np \tag{2.1.3}$$

The power transfer equation for the transformer is given by:

$$Vp * Ip = Vs * Is \tag{2.1.4}$$

Power system transformers are of two types:

a) Power transformers- These are mainly delta/star connected, or star/star connected.
b) Distribution transformers- These are mainly delta/star connected.

Transformers are key components in power systems and power plants. They are also widely applied to uninterruptible power supply. Security and stability of transformers are both important and necessary to system operation. The large transient current of transformers due to flux saturation in the core often causes the malfunction of the protective relaying system, costing time and money as the engineers have to examine closely the transformer and the protective system, to check for faults. The large transient current also causes serious electromagnetic stress impact and shortens the life of transformers [6].

2.2 Losses in a Transformer:

Transformers dissipate energy is in the windings, core, and surrounding structures. Larger transformers are generally more efficient, and those rated for electricity distribution should have efficiency around 98 to 99 per cent. Losses in transformers vary with load current, and may be expressed as "no-load" or "full-load" loss or “per cent of full load loss”. Winding resistance dominates load losses, whereas hysteresis and eddy currents losses contribute to over 99% of the no-load loss. The no-load loss can be significant, so that even an idle transformer constitutes a drain on the electrical supply and a running cost. Designing transformers for lower loss requires a good design of core, good-quality silicon steel, or even amorphous steel for the core and thicker wire, increasing initial cost so that there is a trade-off between initial costs and running cost .The major losses in a transformer are copper loss and iron loss or no-load loss.

The different types of losses are as follows:

a) **Copper loss:** Current flowing through the windings causes resistive heating of the conductors. At higher frequencies, skin effect and proximity effect create additional winding resistance and losses. This contributes to the major percentage of loss during load and zero during no load operation.

b) **Hysteresis loss:** Each time the magnetic field is reversed, a small amount of energy is lost due to hysteresis within the core. For a given core material, the loss is proportional to the frequency, and is a function of the peak flux density to which it is subjected. The Hysteresis loss is given by:

$$P_h = \sigma_h * f * B_m^n * M \tag{2.2.1}$$

Where f is the power system frequency, M is the mass of the core, B_m is the magnetic flux density.

c) **Eddy current loss:** Eddy currents circulate within the core in a plane normal to the flux, and are responsible for resistive heating of the core material. The eddy current loss is a complex function of the square of supply frequency and Inverse Square of the material thickness. Eddy current losses can be reduced by making the core of a stack of plates electrically insulated from each other, rather than a solid block; all transformers operating at low frequencies use laminated or similar cores. The eddy current loss is given by:

$$P_e = \sigma_e * f^2 * B_m^2 * M \tag{2.2.2}$$

Where f is the power system frequency, M is the mass of the core, B_m is the magnetic flux density.

d) **Magnetostriction:** This is relative change of dimensions due to magnetisation of a magnetic material in a magnetic field. Flux in a ferromagnetic material, such as the core, causes it to physically expand and contract slightly with each cycle of the magnetic field. This produces the buzzing sound commonly associated with transformers that can cause losses due to frictional heating. This buzzing is particularly familiar from low-frequency (50 Hz or 60 Hz) mains hum. This loss is directly proportional to the square of the flux density.

e) **Cross-fluxing:** The flux passing between adjacent laminations sets up magnetic forces which cause the lamination to vibrate at twice the supply.

f) **Stray losses:** Leakage inductance is almost lossless, because energy supplied to its magnetic fields is returned to the supply with the next half-cycle. However, any leakage flux that intercepts nearby conductive materials such as the transformer's support structure will give rise to eddy currents and be converted to heat. There are also radiated losses due to the oscillating magnetic field but these are usually small. These are mainly found in the windings and in the transformer tank.

2.3 Transformer Derating:

Transformers are basically designed and built for use at rated frequency and prefect sinusoidal load current. Non-linear loads on a transformer lead to higher losses, early damage of insulation, premature failure and thus reducing the useful life of the transformer. To prevent these problems, the rated capacity of a transformer, which supplies non-linear loads, must be reduced. This is called derating of transformer.

Nowadays, electricity distribution companies are concerned about assigning ratings to transformers for nonsinusoidal load current operation. The envisaged mass production of electric vehicles in future decades may lead to increase non-linear domestic loads due to the large number of battery chargers. Uses of other nonlinear domestic loads such as variable speed thermal pumps are increasing. In addition, due to the widespread use of non-linear loads such as computers, variable speed drives in heating, ventilation and air conditioning (HVAC) systems and electronic ballasts of fluorescent lamps, harmonic distortion is increasing in the commercial user and services [6].

Additional load losses due to non-sinusoidal voltage yield higher hot spot temperatures in transformers. The temperature rise of transformers due to non-sinusoidal load currents was discussed in the IEEE transformer committee meeting, in March 1980. This meeting recommended providing a standard guide for estimation of the loading capacity of the transformers with distorted currents. Kline presented a procedure in which the eddy current losses vary with the square of the current and harmonic order. Finally, an IEEE C57-110 entitled "Recommended practice for establishing transformer capability when supplying nonsinusoidal load currents" has been published. The aim in publishing this standard was to provide a procedure for the determination of the capacity of a transformer under non-sinusoidal loads.

To estimate the de-rating of the transformer, the load's K-factor may be used. This factor is calculated according to the harmonic spectrum of the load current and is an indication of the additional eddy current load losses. It reflects the excess losses experienced in a traditional wire wound transformer.

Modern transformers use alternative winding designs such as foil windings or mixed wire/foil windings. For these transformers, the standardised K-factor – derived for the load current - does not reflect the additional load losses and the actual increase in losses proves to be very dependent on the construction method. It is therefore necessary to minimise the additional losses at the design stage of the transformer for the given load data using field simulation methods or measuring techniques.

2.4 K-factor:

When a non-linear load is supplied from a transformer, it is sometimes necessary to derate the transformer capacity to avoid overheating and subsequent insulation failure. The reason for this is that the increased eddy currents caused by the harmonics increase transformer losses and thus generate additional heat. Also, the RMS load current could be much higher than the kVA rating of the load would indicate. Hence, a transformer rated for the expected load will have insufficient capacity.

The K-Factor is used by transformer manufacturers and their customers to adjust the load rating as a function of the harmonic currents caused by the load(s).

K-factor is a weighting of the harmonic load currents according to their effects on transformer heating, as derived from ANSI/IEEE C57.110. A K-factor of 1.0 indicates a linear load (no harmonics). The higher the K-factor, the greater the harmonic heating effects [7].

There are different ways of measuring K factor. The first, derived by transformer manufacturers in conjunction with Underwriters Laboratories in the United States, is to calculate the factor increase in eddy current loss and specify a transformer designed to cope; otherwise known as the 'K-factor' and given by:

$$K = \sum_{h=1}^{h=hmax} (h^2 * I_h^2) \tag{2.4.1}$$

Where, h = harmonic number

I_h = the fraction of total rms load current at harmonic number h

Power quality meters mostly read the K-factor of the load current directly. Once the K-factor of the load is known, it is a simple matter to choose a transformer with a higher K-rating from the standard range of 4, 9, 13, 20, 30, 40, 50.

For a pure linear load – one which draws a sinusoidal current – will have a K-factor of unity. A higher K-factor indicates that the eddy current loss in the transformer will be K times the value at the fundamental power frequency. 'K-rated' transformers are therefore designed to have very low eddy current loss at fundamental frequency.

The second method, used in Europe, is to estimate by how much a standard transformer should be de-rated so that the total loss on harmonic load does not exceed the fundamental design loss; this is known as 'factor K'.

This factor is given by:

$$K = \left(1 + \left(\frac{e}{1+e}\right) * \left(\frac{I_1}{I}\right)^2 * \sum_{h=2}^{hmax} (h^q * (\frac{I_h}{I})^2\right)^{0.5} \tag{2.4.2}$$

Where, e = ratio of fundamental frequency eddy current loss to ohmic loss, both at reference temperature

h = harmonic number

I = rms of the sinusoidal current including all harmonics

I_h = magnitude of the h^{th} harmonic

I_1 = magnitude of the fundamental current

q = an exponential constant that is dependent on the type of winding and frequency. Typical values are 1.7 for transformers with round or rectangular cross-section conductors in both windings and 1.5 for those with foil low voltage windings.

Standard K-factor loads:

Underwriters laboratory (UL) recognized the potential safety hazards associated with using standard transformers with nonlinear loads and developed a rating system to indicate the capability of a transformer to handle harmonic loads. The ratings are described in UL1561 and are known as transformer K-factors. K-factor transformers are designed to reduce the heating effects of harmonic currents created by loads like those in the table below. The K-factor rating is an index of the transformer's ability to withstand harmonic content while operating within the temperature limits of its insulating system [7].

Table1: Typical K-Factor loads

LOAD	K-Factor
Electric discharge lighting	K-4
UPS with optional input filtering	K-4
Welders	K-4
Induction heating equipment	K-4
PLCs and solid state controls	K-4
Telecommunications equipment	K-13
UPS without input filtering	K-13
Multi wire receptacle circuits in general care areas	K-13
equipment on an assembly or production line	K-13
Mainframe computer loads	K-20
Solid state motor drives	K-20
Multi wire receptacle circuits in critical care areas	K-20

2.5 Analysing Power System Harmonics:

Power systems are designed to operate at frequencies of 50Hz. However, certain types of loads produce currents and voltages with frequencies that are integer multiples of the 50Hz fundamental frequency. These higher frequencies are a form of electrical pollution known as power system harmonics. Harmonics are periodic phenomena that produce continuous distortion of voltage and current waveforms. These periodic nonsinusoidal waveforms are described in terms of their harmonics, and their magnitudes and phase angles are computed using Fourier analysis [8].

Fourier analysis permits a periodic distorted waveform to be decomposed into a series containing dc, fundamental frequency (e.g. 50Hz), second harmonic (e.g. 100Hz), third harmonic (e.g. 150Hz), and so on. The individual harmonics add to reproduce the original waveform. The highest harmonic of interest in power systems is usually the 25th (1250Hz).

Ordinarily, the DC term is not present in power systems because transformers block the flow of DC. Even-ordered harmonics are generally much smaller than odd-ordered harmonics because most electronic loads have the property of half wave symmetry, and half-wave symmetric waveforms have no even-ordered harmonics.

The Fourier series of any physically realizable periodic waveform is given by:

$$i(t) = I_{avg} + \sum_{k=1}^{\infty} I_k \sin(K\omega_1 t + \theta_k) \qquad (2.5.1)$$

I_{avg} is the average (referred to as the "DC" value). I_k are peak magnitudes of the individual harmonics, ω_1 is the fundamental frequency (in radians per second), and θ_k are the harmonic phase angles.

The time period of the waveform is given by:

$$T=2\pi/\omega_1=1/f_1 \qquad (2.5.2)$$

If function $i(t)$ is periodic with an identifiable period T (i.e., i(t) = i(t + NT)), then i(t) can be written in rectangular form as:

$$i(t) = I_{avg} + \sum_{k=1}^{\infty} a_k \cos(k\,\omega_1 t) + b_k \sin(k\,\omega_1 t)) \quad (2.5.3)$$

Where,

$$I_{avg} = \frac{1}{T} * \int_{t_0}^{t_0+T} i(t)dt \quad (2.5.4)$$

$$a_k = \frac{2}{T} * \int_{t_0}^{t_0+T} i(t)\cos(k\,\omega_1 t)dt \quad (2.5.5)$$

$$b_{k\,=}\ \frac{2}{T} * \int_{t_0}^{t_0+T} i(t)\sin(k\,\omega_1 t)dt \quad (2.5.6)$$

If the wave form has odd wave symmetry i,e i(t)= -i(-t), then the corresponding Fourier series does not have any cosine terms. Thus $a_k = 0$ and the other coefficient is found out by doubling the value and integrating over the first half period only.

$$b_{k=} \frac{4}{T} * \int_{0}^{T/2} i(t)\sin(k\,\omega_1 t)dt \quad (2.5.7)$$

If the wave form has even wave symmetry i,e i(t)= i(-t), then the corresponding Fourier series does not have any sine terms. Thus $b_k = 0$ and the other coefficient is found out by doubling the value and integrating over the first half period only.

$$a_{k=} \frac{4}{T} * \int_{0}^{T/2} i(t)\cos(k\,\omega_1 t)dt \quad (2.5.8)$$

If the wave form has half wave symmetry i,e i(t+T/2)= -i(t), then the corresponding Fourier series does not have any even harmonics. Thus b_k and a_k are found out by doubling the value and integrating over the first half period only.

$$a_k = \frac{4}{T} * \int_{0}^{T/2} i(t)\cos(k\,\omega_1 t)dt \quad (2.5.9)$$

$b_k = \frac{4}{T} * \int_0^{T/2} i(t)\sin(k\,\omega_1 t)dt$, for k is odd (2.5.10)

2.6 Total Harmonic Distortion (THD):

The most commonly-used measure for harmonics is total harmonic distortion (THD). THD is defined as the rms value of the harmonics above fundamental, divided by the rms values of the fundamental. This is given by:

THD = Rms value of harmonic component/ RMS value of fundamental component

$$THD = (I_s^2 - I_1^2)^{0.5} / I_1 \quad (2.6.1)$$

Where, I_s is the supply current, I_1 is the fundamental current

This is also given as:

$$THD = \frac{\left(0.5 * \sum_{k=2}^{\infty} I_k^2\right)^{0.5}}{(I_1/(2)^{0.5})} \quad (2.6.2)$$

Where, I_k is the peak value of k^{th} harmonic component.

The RMS value of any periodic waveform is given by:

$$I_{rms}^2 = 0.5 * \sum_{k=1}^{\infty} I_k^2 \quad (2.6.3)$$

Where, I_k is the peak value of k^{th} harmonic component

It is the summation of the squares of the indivisual frequency component RMS values'

$$I_{rms}^2 = I_{1,rms}^2 + I_{2,rms}^2 + I_{3,rms}^2 + \cdots + I_{k,rms}^2 + \cdots \quad (2.6.4)$$

Where $I_{rk,ms}^2$ is the rms value of the k^{th} harmonic component.

The RMS value and the total harmonic distortion for any periodic quantity is given by:

$$I_{rms} = I_{1,rms} * (1 + THD^2)^{0.5} \tag{2.6.5}$$

Current distortion in loads varies from a few per cent to more than 100%, but and those greater than 10% are definitely unacceptable and will cause problems for sensitive equipment and loads [8].

Chapter **3**

Equivalent Circuit Model of Transformer

3.1 Equivalent Circuit of Transformer:

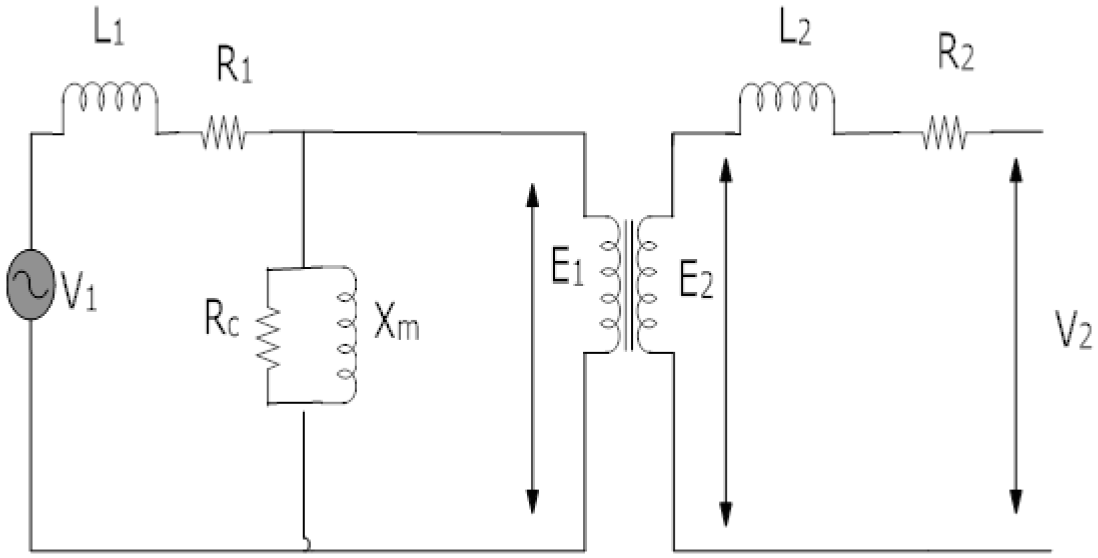

Fig.1. equivalent circuit of transformer

Equivalent circuit for a single phase transformer with N_P turns in the primary winding and N_S in the secondary, showing:

- Primary winding resistance R_1
- Primary leakage inductance L_1
- Secondary winding resistance R_2
- Secondary leakage inductance L_2
- Equivalent core loss resistance R_c
- Magnetising reactance X_m
- Primary and secondary EMF E_1 and E_2, developed over an ideal transformer
- Primary and secondary terminal voltages V_1 and V_2

The equivalent circuit drawn above is called the exact equivalent circuit of the transformer. The resistances of the windings are given by R_1 and R_2 of primary and secondary respectively. The leakage inductances are given by L_1 and L_2 of primary and secondary respectively. The core loss component is given by R_c. This is a virtual resistance which represents the iron losses of the transformer and X_m represents the magnetising branch. It decides the amount of current required to magnetise the core. The current flowing through it is I_m and the current flowing through the core loss component is I_c. These two components are orthogonal to each other. The combination of these two currents provides the no load component. The supply of the transformer provides the no load component as well as the load current when it is connected to any load.

The equivalent circuit model above is a bit complicated while solving for various quantities unknown in the transformer. The secondary side has to be solved and then its value is reflected to the primary side and hence the unknown parameters are solved. Thus it would be easier if in the model, the secondary side parameters are reflected to the primary side by the transformation ratio. Thus the secondary side just behaves as if a ideal transformer with no resistance and leakage reactance. The secondary side resistance is reflected to the primary side by multiplying the secondary side resistance with the square of the turns ratio. Similarly the leakage reactance is reflected to the primary side by the same technique. Thus the referred quantities to the primary side are given by:

$$R_2' = R_2 * (N_p/N_s)^2 \tag{3.1.1}$$

$$X_2' = X_2 * (N_p/N_s)^2 \tag{3.1.2}$$

The resulting model is sometimes termed the exact equivalent circuit, though it retains a number of approximations. Analysis is again simplified by moving the magnetizing branch to the left of the primary impedance, an implicit assumption that the magnetizing current is low, and then summing primary and referred secondary impedances, resulting in so-called equivalent impedance. This assumption is based on the fact that the no load current is 2 to 5 per cent of the full load current. The shunt branch is moved across the primary impedance and kept in parallel with the voltage supply. Thus the final exact equivalent circuit model is given below:

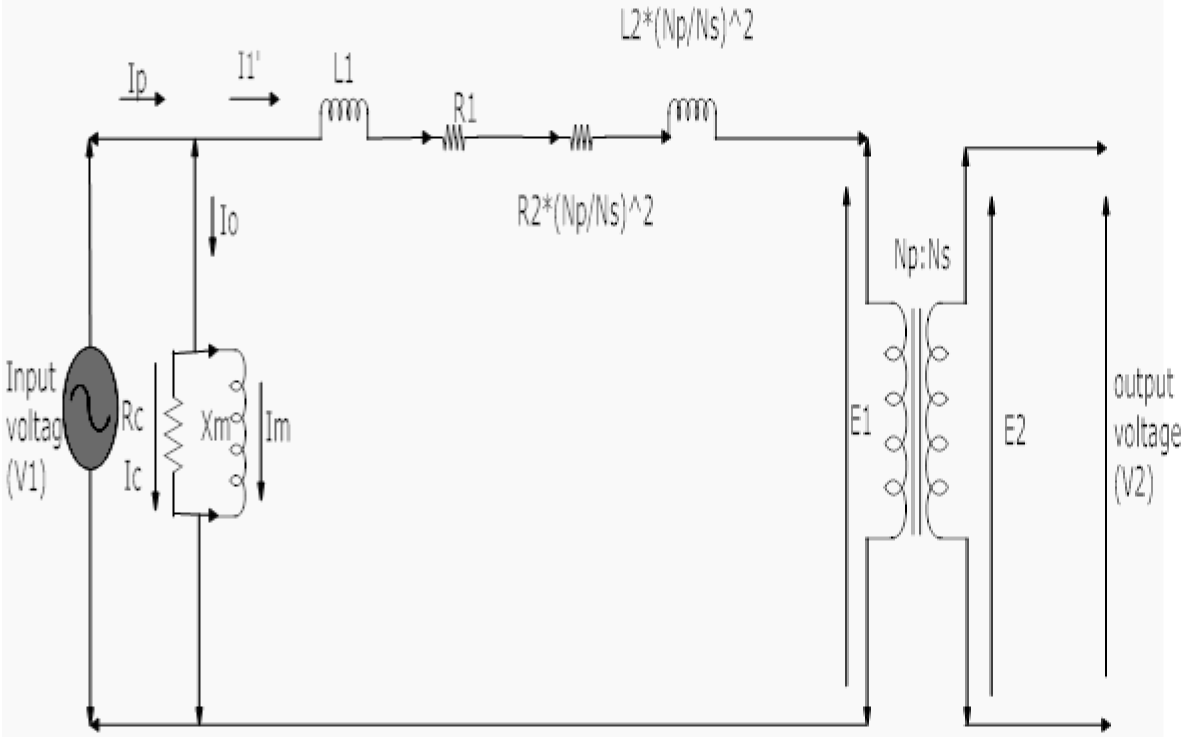

Fig.2. equivalent circuit of transformer with secondary parameters referred to primary

Where,

- Primary winding resistance R_1
- Primary leakage inductance L_1
- Secondary winding resistance R_2, referred to the primary circuit by the turns ratio squared
- Secondary leakage inductance X_2, referred to the primary circuit by the turns ratio squared
- Equivalent core loss resistance R_c and core loss current I_c
- Magnetising reactance X_m and magnetising current I_m
- No-load current I_0
- Primary and secondary EMF E_1 and E_2, developed over an ideal transformer
- Primary and secondary terminal voltages V_1 and V_2 .
- Primary terminal current I_p.

3.2 Saturation of Transformer:

Saturation of transformer may occur due to many reasons, some of which are listed as below [9]:

a) Normal excitation: Even under normal excitation condition, transformer core may have entered, slightly, the saturation region and begin to generate some harmonics in the excitation current. The degree of the saturation depends on the transformer design.

b) Over excitation: Over excitation is basically caused by over voltages. This problem is particularly subjected in the case of transformers connected to large rectifier plant following load rejection. Over voltage shifts the peak operation point of the transformer excitation characteristics up to saturation region so that different harmonics are generated. The magnetizing current of over excitation is often symmetrical.

c) Converter load: Converter loads may draw DC and low frequency currents from supplying transformers. The transformer cores are biased by these load currents and driven to saturation by these harmonics components. Ex, a cycloconvertor connected to a single phase load will draw dc currents from the transformer as well as harmonic components which are integral multiples of two times the supply frequency.

d) Geomagnetically induced currents: Geomagnetically Induced Currents (GIC) flow on the earth surface due to Geomagnetic Disturbance (GMD). They are typically 0.001 to 0.1 Hz and could reach peak values as high as 200A. They enter transformer windings by way of grounded wye connections and bias the transformer cores to cause half cycle saturation.

3.3 Effect of saturation with sinusoidal Supply Voltage:

To analyse the excitation current the equivalent circuit of an unloaded single phase transformer was used. A sinusoidal supply voltage was given to the single phase saturable transformer. The circuit included an ammeter and a voltmeter to measure current and voltage respectively. The meters are connected to the oscilloscope to view waveforms. The Simulink model for the setup is given below. The observed excitation current waveform for the sinusoidal voltage is obtained. The fast Fourier analysis of the excitation current is done and the total harmonic distortion for various components is calculated. The equivalent circuit of transformer under no load is given below.

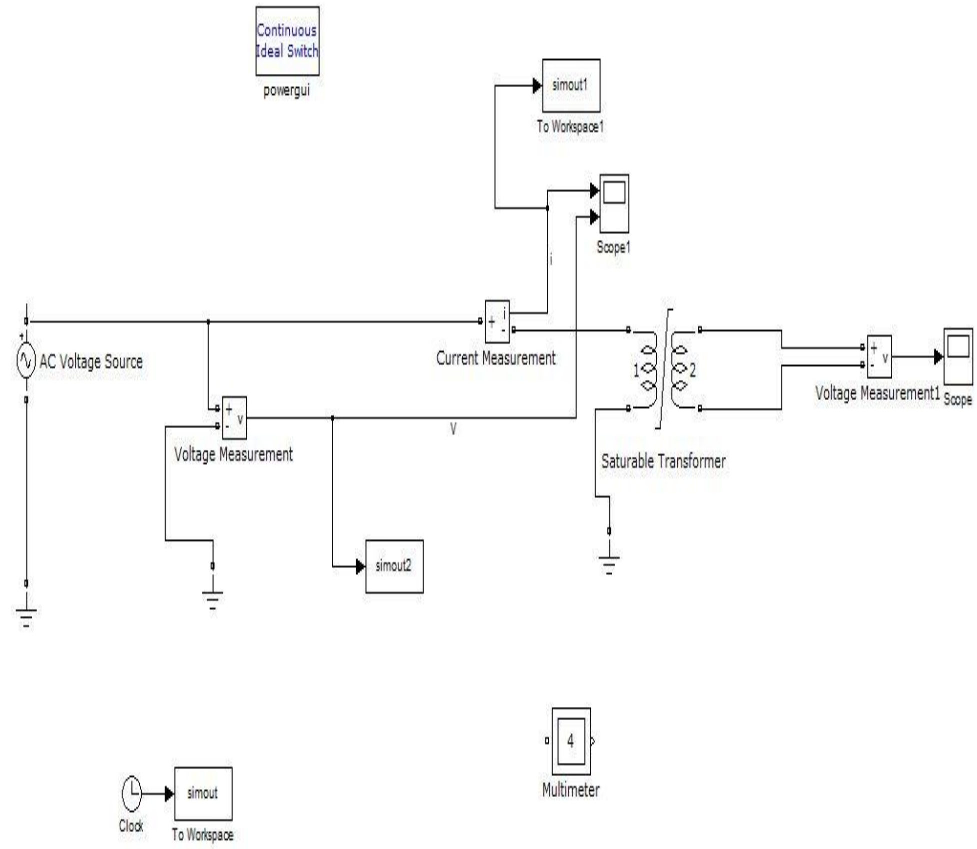

Fig.3. representation of Simulink model of transformer under sinusoidal excitation

3.4 Effect of saturation with non-sinusoidal Supply Voltage:

To analyse the excitation current the equivalent circuit of an unloaded single phase transformer was used. A non-sinusoidal supply voltage was given to the single phase saturable transformer. The non-sinusoidal supply voltage is created by summing up different voltage sources with different magnitude and frequency of each voltage source is an integral multiple of the fundamental. The circuit included an ammeter and a voltmeter to measure current and voltage respectively. The meters are connected to the oscilloscope to view waveforms. The Simulink model for the setup is given below. The observed excitation current waveform for the sinusoidal voltage is obtained. The fast Fourier analysis of the excitation current is done and the total harmonic distortion for various components is calculated. The equivalent circuit of transformer under no load is given below. The AC voltage source has the following sequence:

$AC\ Voltage\ source = A \sin(\omega_1 t)$

$AC\ Voltage\ source1 = A_1 \sin(3\,\omega_1 t)$

$AC\ Voltage\ source2 = A_2 \sin(5\,\omega_1 t)$

$AC\ Voltage\ source3 = A_3 \sin(7\,\omega_1 t)$

Where, $\omega_1 = 2 * 3.14 * f$, F if the fundamental frequency of the system (50 Hz)

Here even harmonics are neglected as they are usually absent in the power system. The voltage waveforms are also assumed to have half wave symmetry which results in the redundancy of the 2^{nd} order harmonics. The 2^{nd} order harmonics are present in case of magnetic inrush current only during switching of the transformer. This magnetic inrush current is a transient phenomenon and reduces quickly in 2 to 4 cycles. The harmonic spectrum of inrush current gives the 2^{nd} order harmonics.

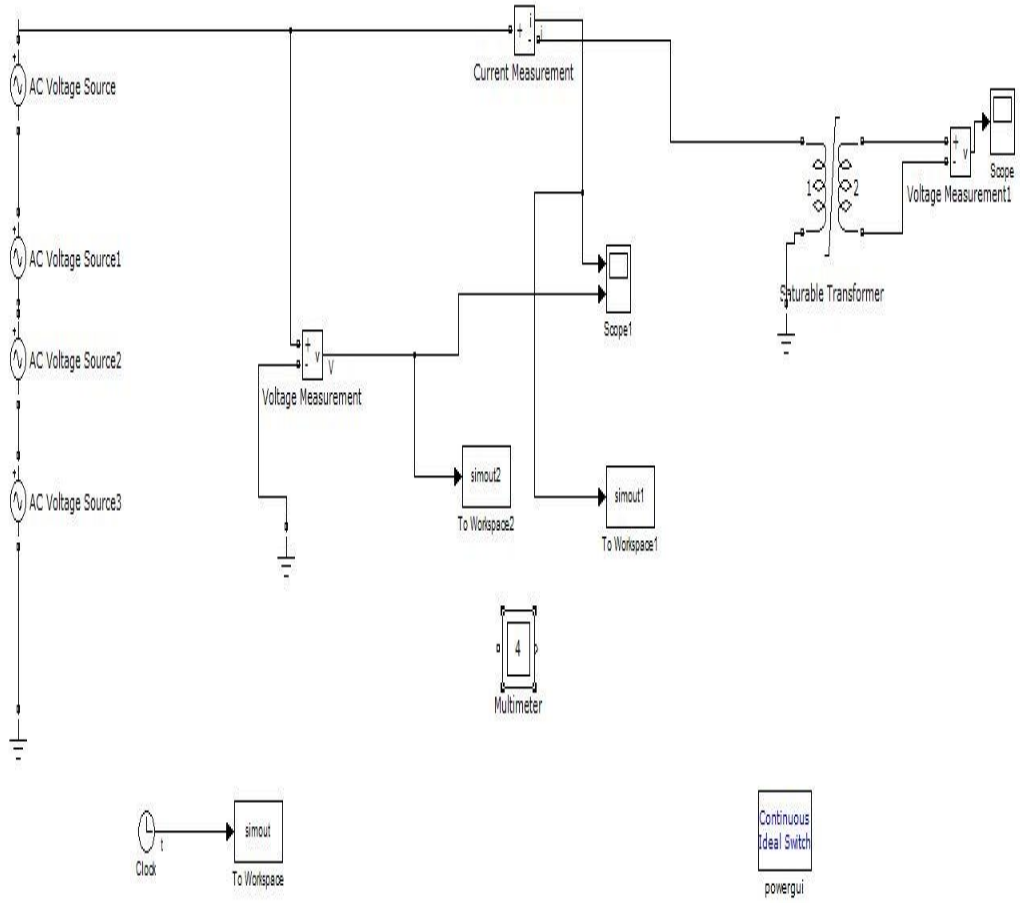

Fig.4. representation of Simulink model of transformer under nonsinusoidal excitation

3.5 Modelling of hysteresis loss:

The flux density B(t) is taken to be sinusoidal, which is an odd periodic function. The hysteresis loss is calculated from the hysteresis curve of the laminated steel provided by the manufacturer.

The flux density function is odd and (1/f) periodic. It has a maximum at $t_{max} = 1/(4f)$ and the derivative of the flux density at other points in $[0, \frac{1}{2f}]$ is not zero.

The eddy current loss is augmented into the hysteresis loop by the equation by:

$$H_{new}(t) = H(t) + [\varepsilon \times (dB/dt)] \tag{3.5.1}$$

On integrating the $H_{new}(t)$ over the flux density from t= $[0, \frac{1}{f}]$ we get the total core losses of transformer:

$$P_{total} = f * \oint H_{new}(t) \partial B \tag{3.5.2}$$

The eddy current loss is given by:

$$P_{eddy} = f * \in * \int_0^{1/f} \left(\frac{dB}{dt}\right) * dB(t) \tag{3.5.3}$$

$$P_{eddy} = f * \in * \int_0^{\frac{1}{f}} \left(\frac{dB}{dt}\right)^2 dt \tag{3.5.4}$$

This is also given by:

$$P_{eddy} = 2 * \pi^2 * B_m^2 * f^2 \tag{3.5.5}$$

The hysteresis loss is given by the difference of equations 3.5.2 and 3.5.3 . This value can be re checked by equating it to the value of equation given by:

$$P_{hys} = f * \oint H(t) dB(t) \tag{3.5.6}$$

3.6 Modelling of Transformer without Core Saturation:

There are two ways of modelling the transformer. The 1st one is based on the analysis of the electromagnetic field of the transformer. The 2nd one is the use of lumped circuit model to model a transformer.

In the lumped circuit model, the magnetic behaviour and characteristics are modelled in a single electrical circuit with nonlinear inductors with the nonlinear characteristics being a function of each nonlinear inductor current. From the implementation point of view, the main difficulties of using lumped circuit models are: first, the equivalent circuit will become quite complicated and secondly, the resulting impedance matrices might become ill-conditioned for certain network connections [9].

The transformer model used here is based on the analysis of electromagnetic field in the transformer. In this method the transformer is represented using two separate equivalent circuits, one is the magnetic circuit and other is the electrical circuit.

The modelling equations for the transformer are as follows. The input parameters of the transformer are the input and output voltages, the primary resistances and inductances, the secondary resistances and inductances referred to primary side, the mutual inductance [11].

The input voltage equation is given by:

$$V_1 = i_1 r_1 + \left(\frac{1}{\omega_b}\right) * \left(\frac{d\varphi_1}{dt}\right) \tag{3.6.1}$$

The output voltage equation is given by:

$$V_2 = i_2 r_2 + \left(\frac{1}{\omega_b}\right) * \left(\frac{d\varphi_2}{dt}\right) \tag{3.6.2}$$

Where, $\varphi_1 = \omega_b \lambda_1$, $\varphi_2 = \omega_b \lambda_2$ and ω_b is the base frequency at which the reactance is calculated.

The flux linkage per second of the winding is given by:

$$\varphi_1 = \omega_b \lambda_1 = x_{l1} i_1 + \varphi_m \tag{3.6.3}$$

$$\varphi_2 = \omega_b \lambda_2 = x'_{l2} i'_2 + \varphi_m \tag{3.6.4}$$

Where,

$$\varphi_m = \omega_b l_{m1} (i_1 + i'_2) = x_{m1} (i_1 + i'_2) \tag{3.6.5}$$

The currents in the windings from equations 3.6.3 and 3.6.4 are expressed as:

$$i_1 = (\varphi_1 - \varphi_m) / x_{l1} \tag{3.6.6}$$

$$i'_2 = (\varphi'_2 - \varphi_m) / x'_{l2} \tag{3.6.7}$$

Using equations 3.6.5, 3.6.6 and 3.6.7 we get,

$$(\varphi_m / x_{m1}) = ((\varphi_1 - \varphi_m) / x_{l1}) + ((\varphi'_2 - \varphi_m) / x'_{l2}) \tag{3.6.8}$$

Equation 3.6.8 can also be written as:

$$\varphi_m = x_m [(\varphi_1 / x_{l1}) + (\varphi'_2 / x'_{l2})] \tag{3.6.9}$$

Where, $\left(\frac{1}{x_m}\right) = \left(\frac{1}{x_{m1}}\right) + \left(\frac{1}{x_{l1}}\right) + \left(\frac{1}{x'_{l2}}\right)$ (3.6.10)

From equation 3.6.1,

$$\varphi_1 = \int ((\omega_b V_1) - ((\omega_b r_1) \times (\varphi_1 - \varphi_m) / x_{l1}))\partial t \tag{3.6.11}$$

From equation 3.6.2,

$$\varphi_2 = \int ((\omega_b V''_2) - ((\omega_b r_2) \times (\varphi'_2 - \varphi_m) / x'_{l2}))\partial t \tag{3.6.12}$$

Thus equations 3.6.11, 3.6.12, 3.6.6, 3.6.7 basically forms the equations of transformer model.

The Simulink model of the transformer is given below:

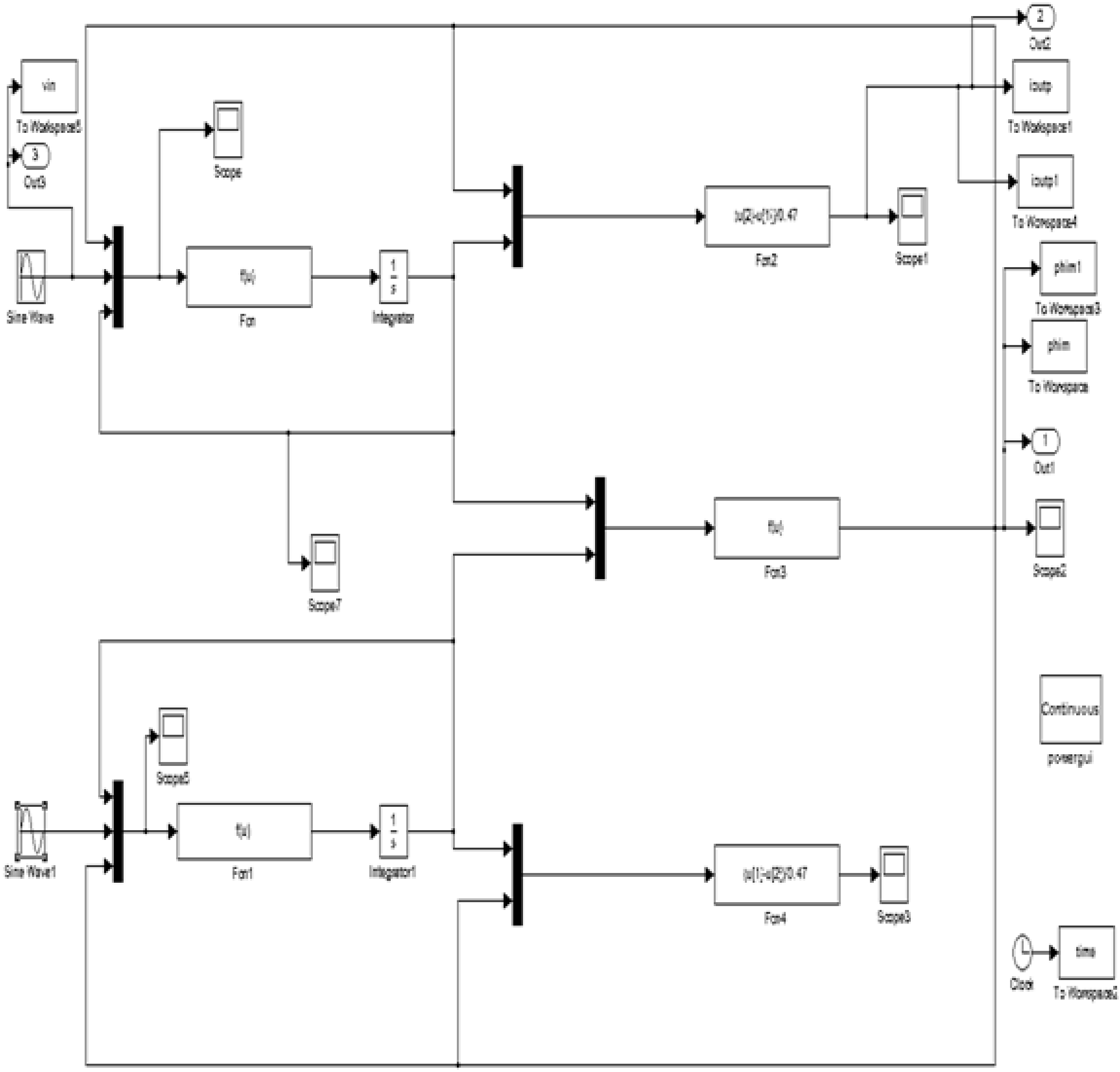

Fig.5. Simulink model of transformer without core saturation

3.7 Modelling of Transformer with Core Saturation:

Core saturation mainly affects the value of mutual inductances rather than the self-inductances and leakage inductances. The mathematical modelling of the leakage inductances is very complicated as it requires constructional details of the core material. The mathematical modelling is thus based on mutual inductances only. Core saturation is obtained from the open circuit magnetisation curve of the transformer which is in fact obtained from the open circuit test of the transformer. The open circuit curve by plotting the open circuit voltage against the no load current of the primary side. From the curve, the region below the knee point is called the unsaturated region where the ration between the terminal voltage and current is constant whereas the above the knee point the ratio gradually decreases.

There are 3 methods to incorporate saturation into the transformer model. These are given by:

a) Using the appropriate value of the mutual reactance's at each time step of the simulation.
b) Approximating the magnetising current by some analytic function of the saturated flux linkage.
c) Using the relationship between the saturated and unsaturated values of the mutual flux linkage [12].

The modelling technique used here is the relationship between the saturated and the unsaturated values of the mutual flux. The relation between the saturated value of flux and the unsaturated value is given as:

$$\varphi_m^{unsat} = \varphi_m^{sat} + \Delta\varphi \qquad (3.7.1)$$

From the saturation curve of the iron core transformer and the air gap magnetisation curve , we can find out the deviation of flux from the saturated flux. This flux is $\Delta\varphi$. This is then plotted against the saturated flux in both positive half and negative half. This serves as an input to the look up table which is used in the Simulink model to incorporate saturation in the core in the mathematical model. Depending upon the instantaneous value of flux a corresponding deviation of flux from saturated is generated from the look up table and this value manipulated with the original flux value to get the mutual flux of the core.

The equations are given as:

$$\varphi_m^{unsat} = \omega_b l_{m1}^{unsat}(i_1 + i_2^{'}) = x_{m1}^{unsat}(i_1 + i_2^{'}) \tag{3.7.2}$$

The saturated values of currents in terms of saturated flux linkage are given by:

$$i_1 = (\varphi_1 - \varphi_m^{sat}) / x_{l1} \tag{3.7.3}$$

$$i_2^{'} = (\varphi_2^{'} - \varphi_m^{sat}) / x_{l2}^{'} \tag{3.7.4}$$

From the equations 3.7.2, 3.7.3, 3.7.4 we get,

$$(\varphi_m^{unsat} / x_{m1}^{unsat}) = ((\varphi_1 - \varphi_m^{sat}) / x_{l1}) + ((\varphi_2^{'} - \varphi_m^{sat}) / x_{l2}^{'}) \tag{3.7.5}$$

Using equation 3.7.1 in the above equation we get,

$$\varphi_m^{sat} = x_m((\varphi_1 / x_{l1}) + (\varphi_2^{'} / x_{l2}^{'}) - (\Delta\varphi / x_{m1}^{unsat})) \tag{3.7.6}$$

Where,

$$\left(\frac{1}{x_m}\right) = \left(\frac{1}{x_{m1}^{unsat}}\right) + \left(\frac{1}{x_{l1}}\right) + \left(\frac{1}{x_{l2}'}\right) \tag{3.7.7}$$

The individual flux equations are given as ,

$$\varphi_1 = \int((\omega_b V_1) - ((\omega_b r_1) \times (\varphi_1 - \varphi_m^{sat}) / x_{l1}))\partial t \tag{3.7.8}$$

$$\varphi_2 = \int((\omega_b V_2^{''}) - ((\omega_b r_2) \times (\varphi_2^{'} - \varphi_m^{sat}) / x_{l2}^{'}))\partial t \tag{3.7.9}$$

Equation 3.7.8, 3.7.9, 3.7.3 and 3.7.4 are the basic equations in modelling the saturation in the core of the transformer.

The Simulink model is given by:

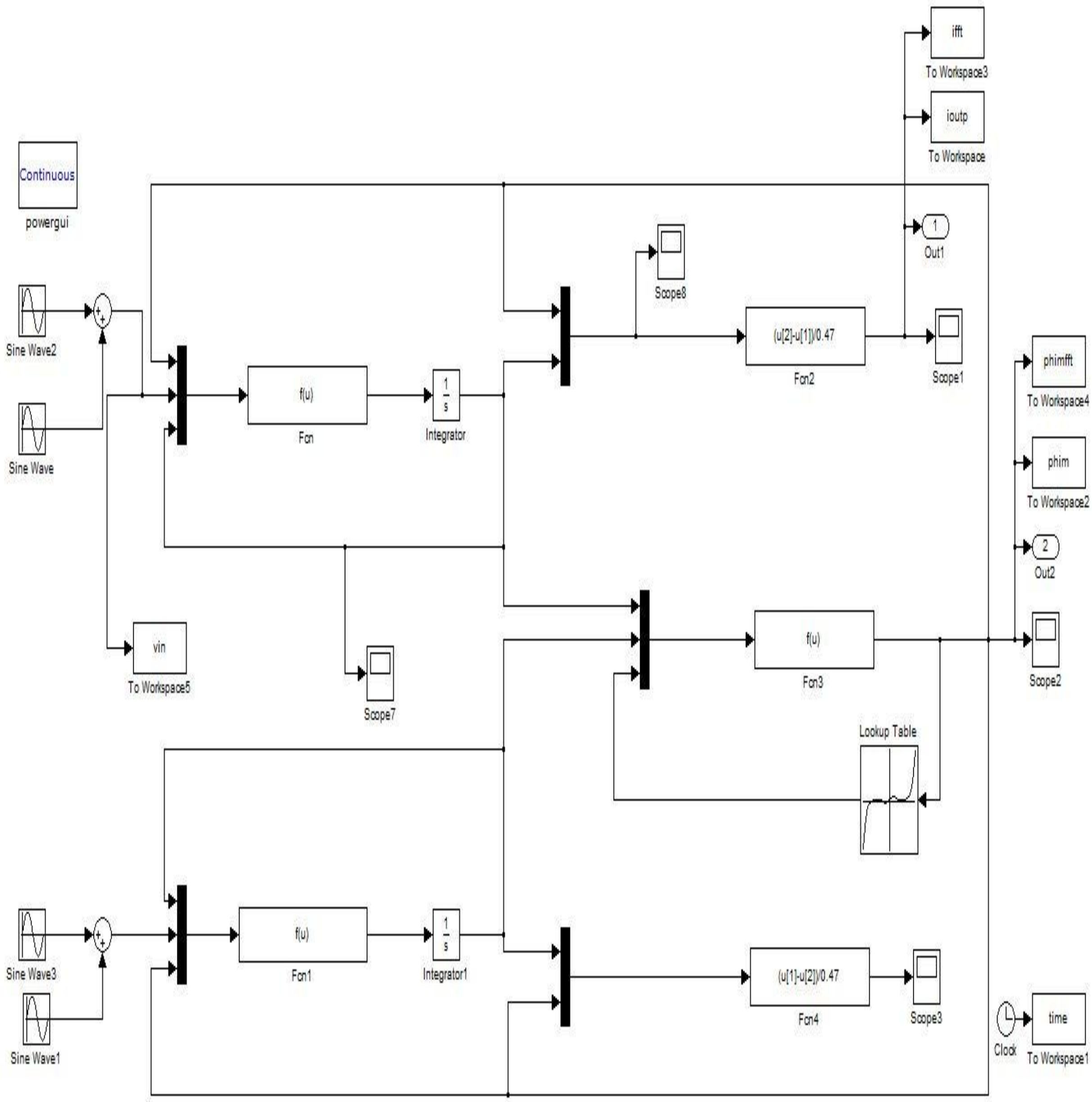

Fig.6. Simulink model representing transformer with core saturation.

Chapter **4**

Simulation and Experimental Results

4.1 Experimental results:

The transformer used in the experiment is a single phase 3 KVA, 230 V/230 V Transformer.

The open circuit test was per formed on the transformer to find the open circuit magnetisation curve of the transformer. The secondary of the transformer was left open circuited. The primary side is connected to voltage source, an ammeter in series with the voltage source and a wattmeter to measure the no load power loss. A low PF wattmeter of 0.2 is used as the no load current has very low PF because it is mainly the magnetising component which is quadratic in nature. The voltages are gradually increased from zero to more than rated value. The open circuit results are as follows:

Table2. Open circuit test results

SL NO	VOLTAGE INPUT (V1) (volts)	CURRENT(mA)	VOLTAGE OUTPUT (V2) (volts)	LOSSES(watts)
1	0	0	0	0
2	30	41	29.2	1.1
3	50	59	48.8	1.5
4	70	75	69	2
5	90	90	89	2.5
6	110	106	106	3.8
7	130	124	125	5.5
8	150	146	145	7.5
9	180	170	169	9.6
10	200	200	194	11.7

11	220	237	219	13.8
12	240	286	240	15.8
13	260	362	259	18
14	280	465	280	20

The short circuit test was done on the primary side of the transformer. The secondary was short circuited and reduced voltage was applied to the primary side to flow rated current in the primary winding. The following results were obtained:

Table3. Short Circuit Test results

SL NO	VOLTAGE(volts)	CURRENT(mA)	LOSSES(watts)
1	21	13	222

From the above open circuit and short circuit tests the parameters of the given transformer was calculated using the derived formulas and the following values are obtained:

$r_1 = 0.6568\Omega,\ r_2' = 0.6568\Omega,\ x_{l1} = 0.47\Omega,\ x_{l2}' = 0.47\Omega$

The above results were used in the mathematical modelling of the transformer.

4.2 Simulation Results and Figures:

There are 3 cases arising from the transformer model. These are the following:

case1: modelling with sinusoidal supply without core saturation.

case2: modelling with sinusoidal supply with core saturation.

case3: modelling with non-sinusoidal supply with core saturation.

a) *With Sinusoidal Supply without Core Saturation:* The input voltage of 230v is given to the transformer model without core saturation.

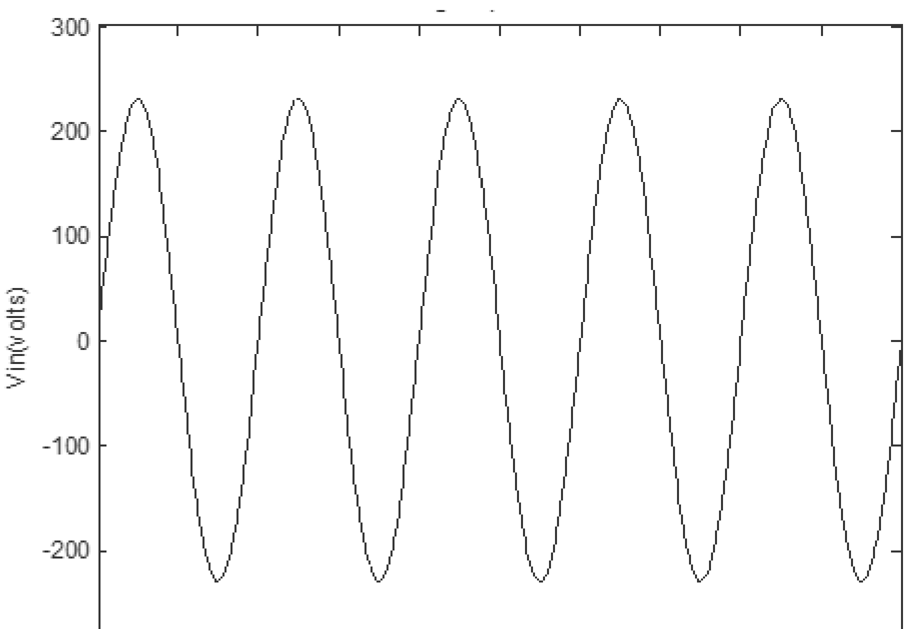

Fig.7. input voltage to the transformer

The waveform below depicts the Excitation Current waveform without core saturation effect. The waveform is almost sinusoidal with peak amplitude of 1.05mA.

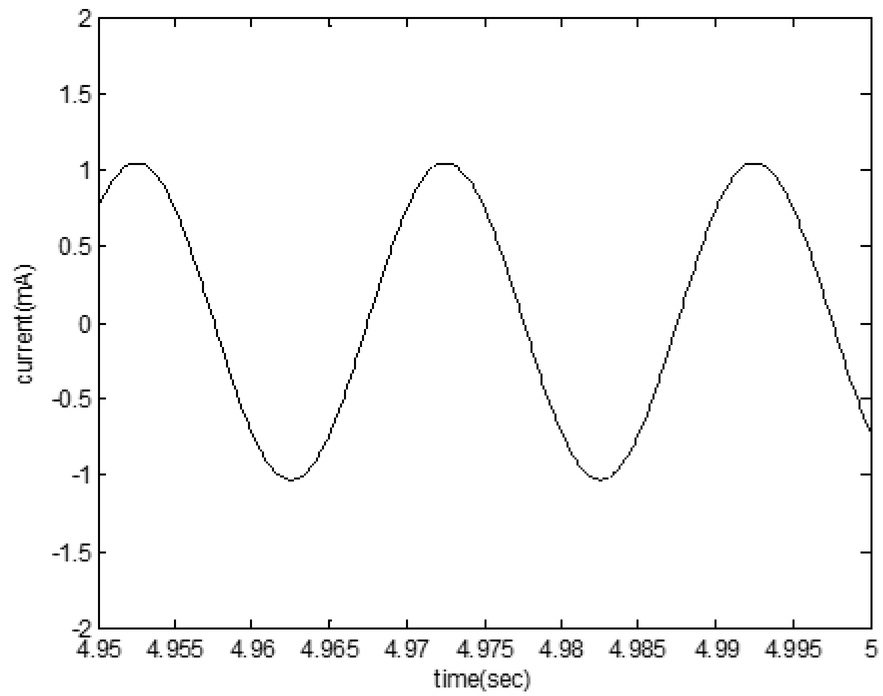

Fig.8. output current waveform

The FFt analysis of the Excitation Current waveform is obtained which contains a small amount of DC component and mainly the fundamental frequency(50 Hz). The THD measured is only 0.05 per cent which shows that the Current is almost Sinusoidal.

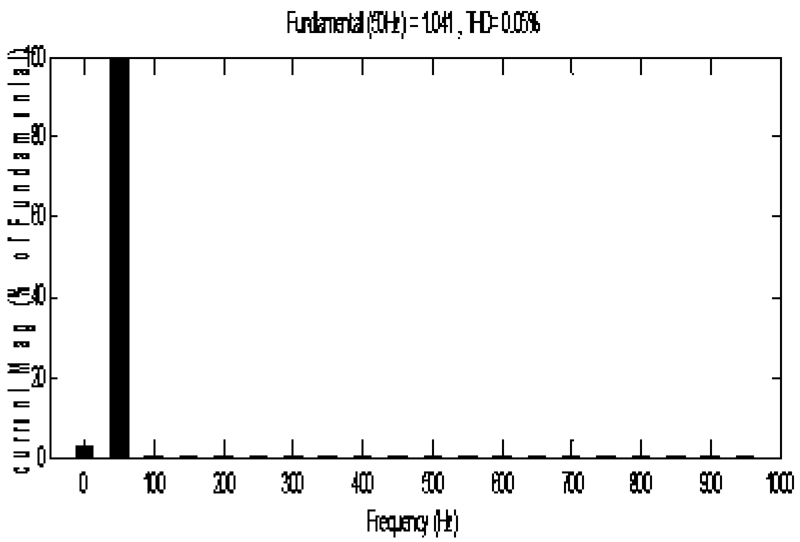

Fig.9. FFT analysis of current waveform

The waveform shown below represents the mutual flux waveform without core saturation. The peak value of the waveform is 230mWb. The flux is sinusoidal following the supply voltage

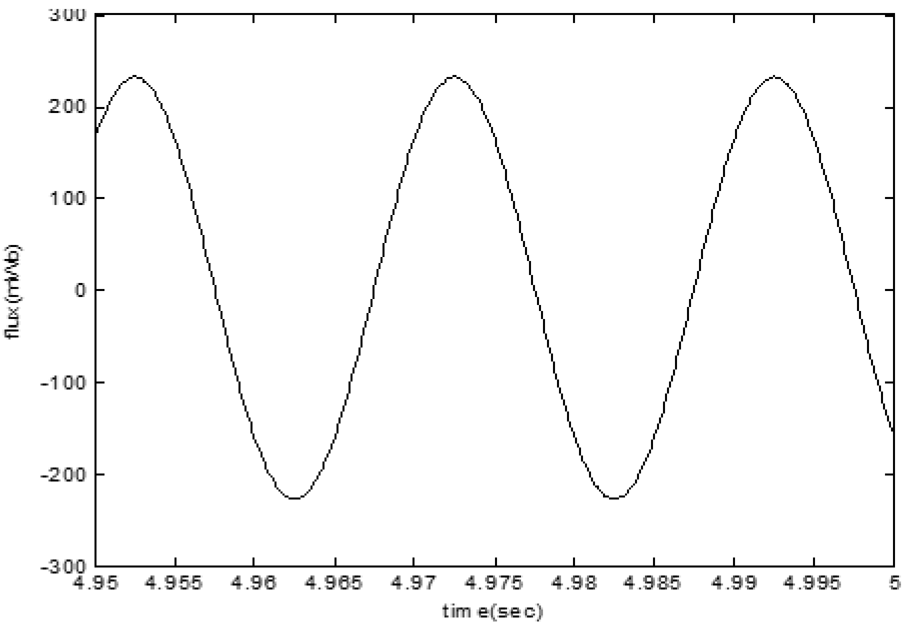

Fig.10. mutual flux wave form

The FFt analysis of the flux waveforms shows that the THD is only 0.05 per cent with the fundamental component having a magnitude of 229.5mWb. There is a small amount of DC component present in the harmonic spectrum.

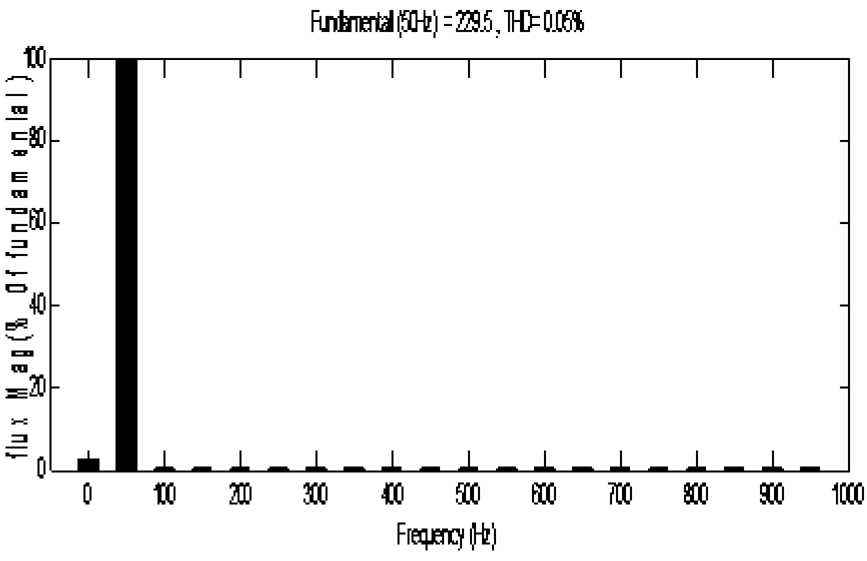

Fig.11. FFT analysis of mutual flux

b) *With Sinusoidal supply with Core Saturation*:

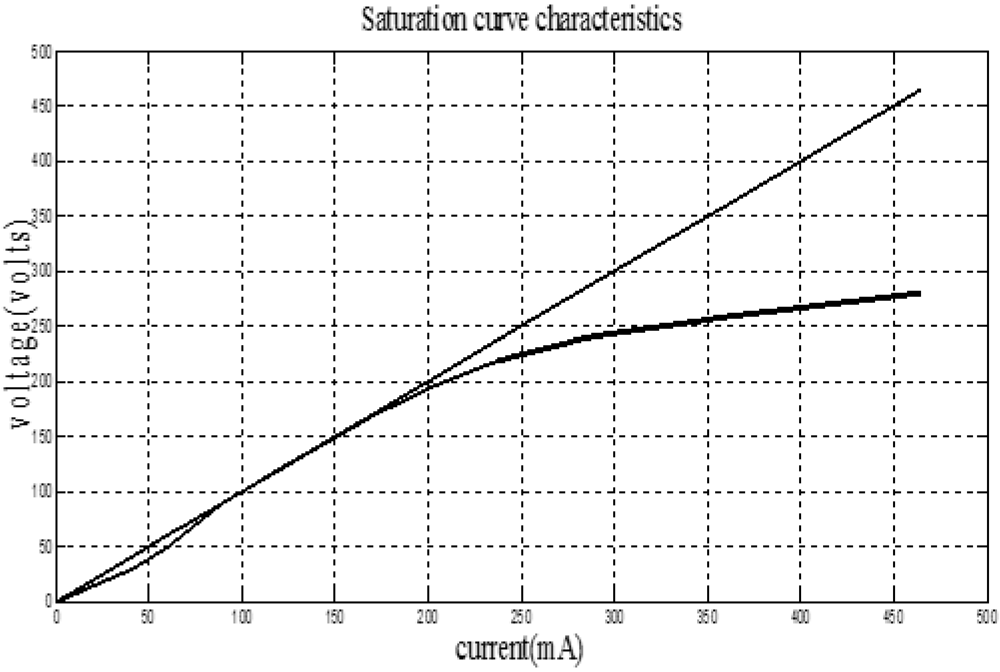

Fig.12. saturation curve from open circuit characteristics

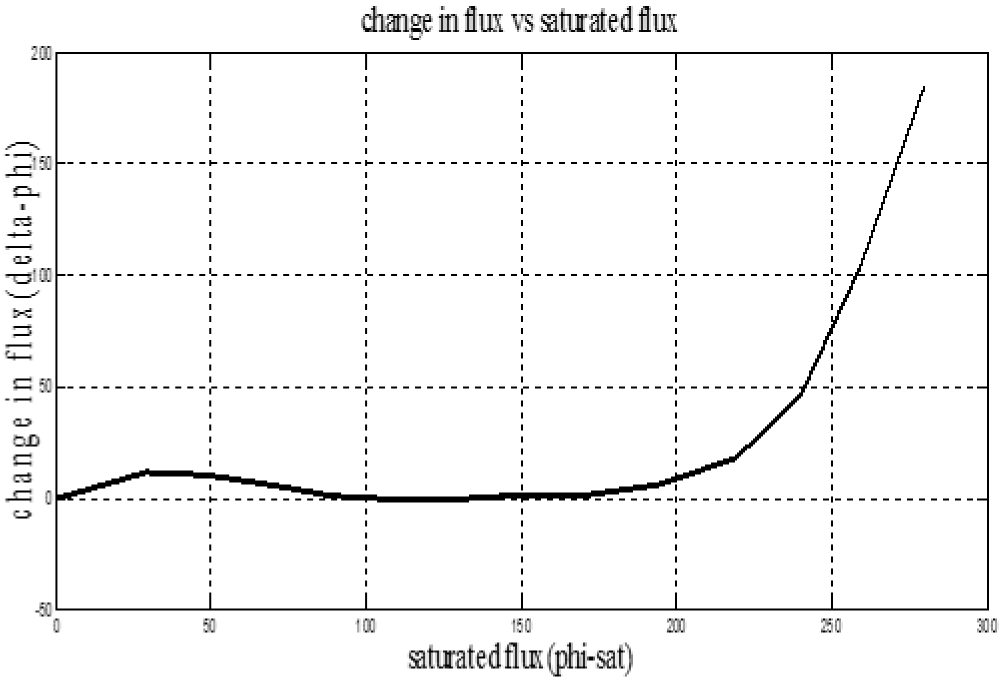

Fig.13. plot of deviation in flux and saturated flux

The waveform given below represents the Excitation current waveform with Sinusoidal input and Core Saturation. The waveforms deviates from the sinusoidal nature due to the saturation effect. The peak value of the current is 6 mA.

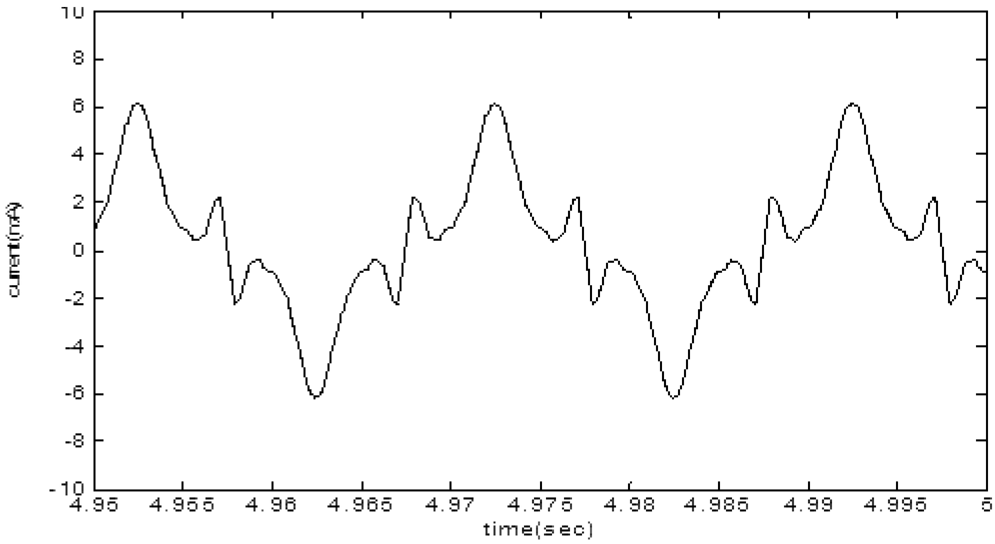

Fig.14. current waveform for saturation with sinusoidal input

The FFT analysis of the waveform shows that there is a THD of 54.92 per cent. The major component is the fundamental one with value 3.593 mA and the other harmonics mainly present are the 3^{rd} harmonic and the 5^{th} harmonic components.

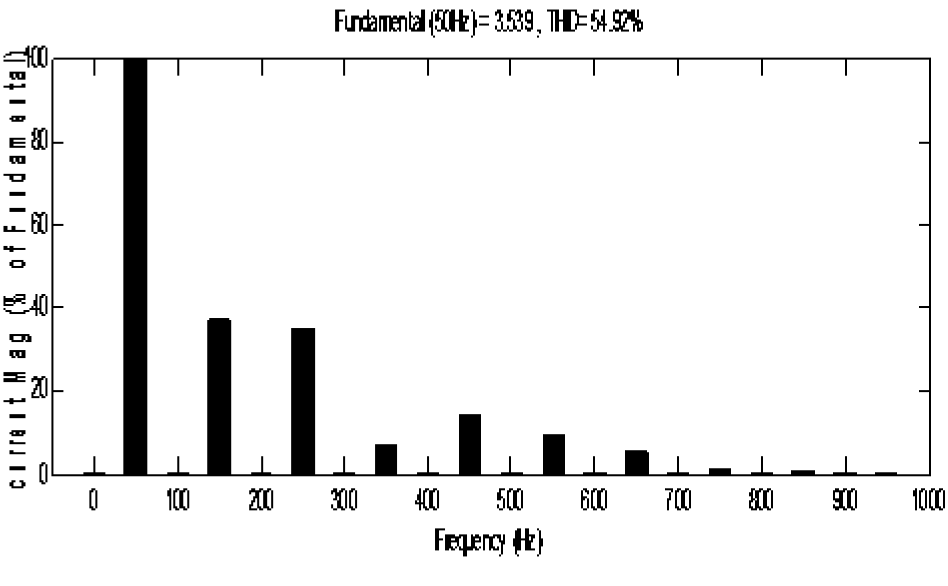

Fig.15. FFT analysis of current waveform

The waveform represents the mutual Flux of transformer model with saturated core. Since the voltage is sinusoidal at 230v the flux is also sinusoidal, and it gets distortred at higher values of supply voltage.

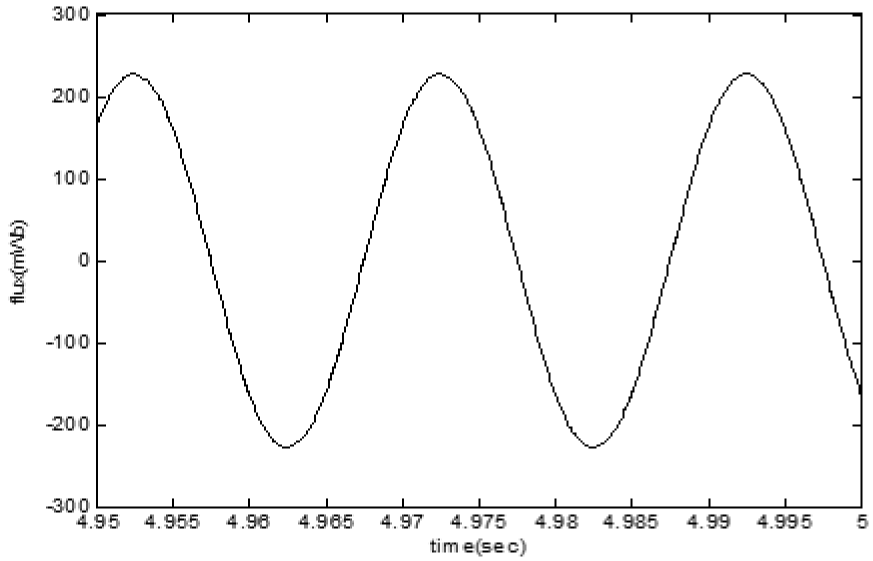

Fig.16. mutual flux waveform

The FFT analysis of the mutual flux waveform shows that the major component is the fundamental one. Its magnitude is 228.3 mWb. The harmonic distortion is 0.43 per cent which clearly reflects that the flux is sinusoidal following the voltage.

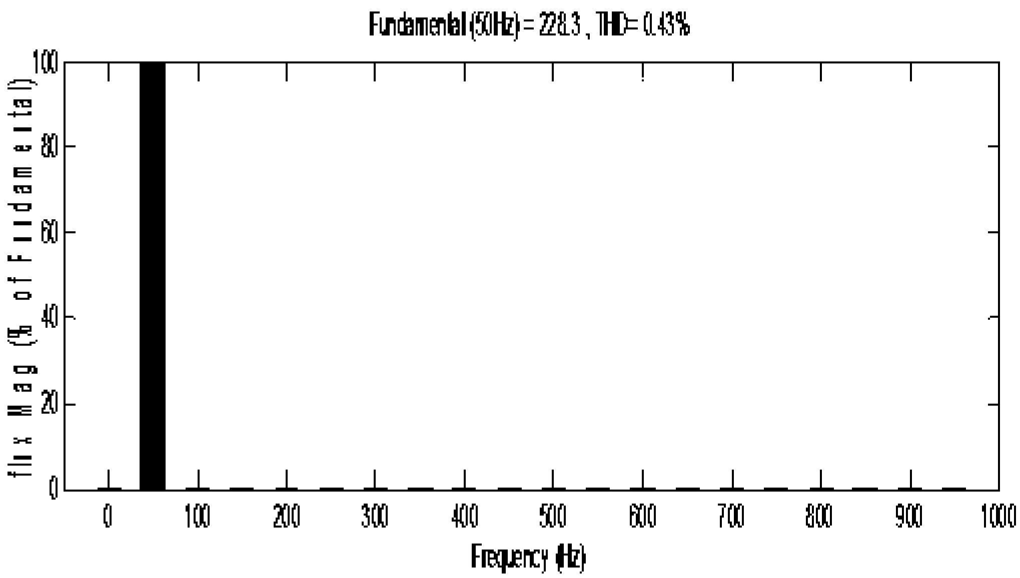

Fig.17. FFT analysis of mutual flux waveform

c) *With Non-sinusoidal input with Core Saturation*: The voltage input to the transformer model without saturation is a non-sinusoidal one due to the injection of the 3^{rd} order harmonic component.

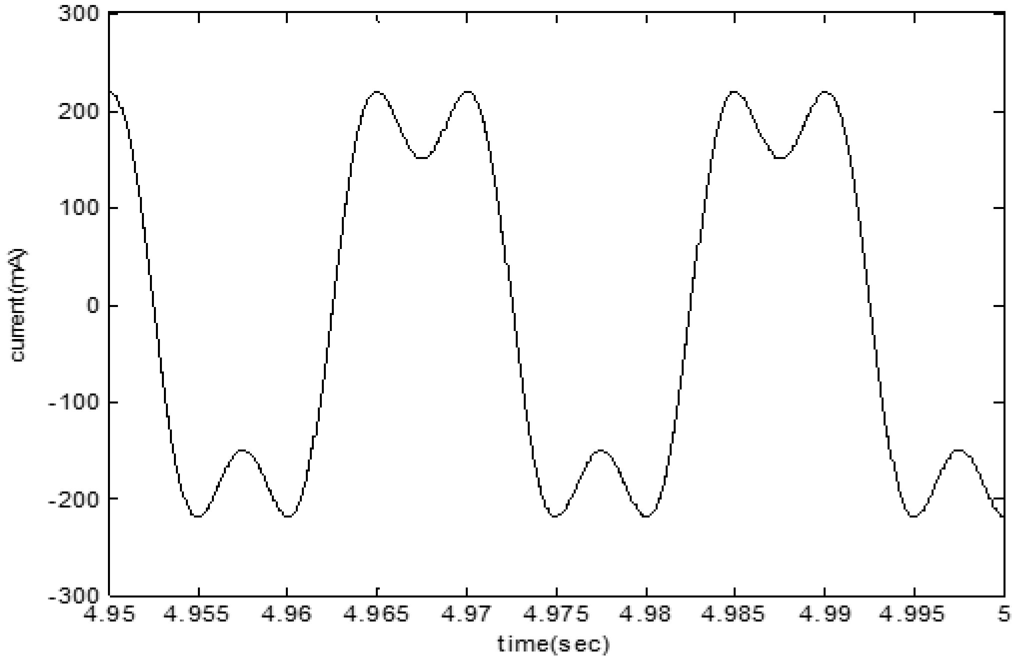

Fig.18. non-sinusoidal input to Transformer

The THD of the input waveform is 34.78% . This is the percentage of the 3^{rd} harmonic component in the Voltage waveform.

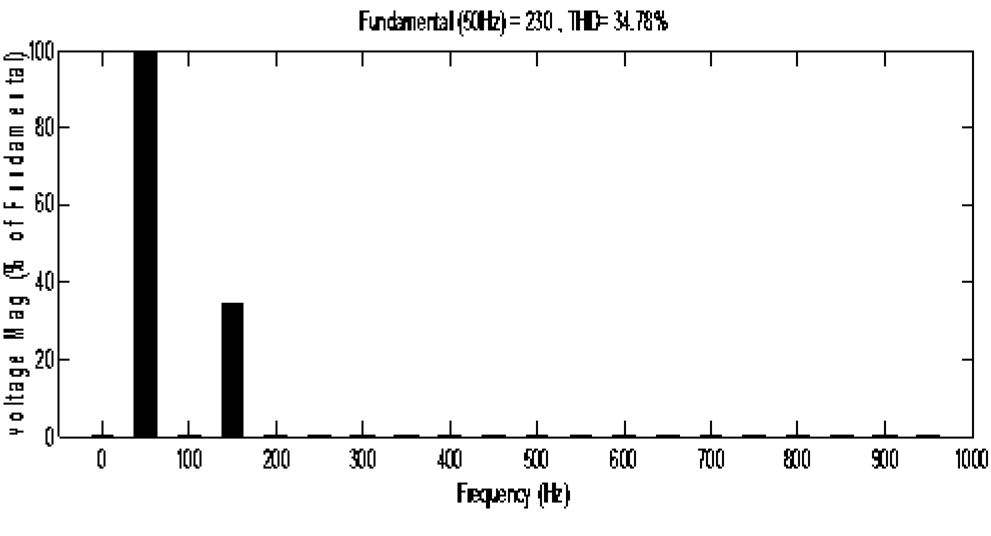

Fig.19. FFT analysis of the non-sinusoidal input voltage

With the injection of the Non-sinusoidal Supply Voltage to the model with Core Saturation, the Excitation current waveform gets more peaky and the peak amplitude is 15mA. The Current also deviates from the sinusoidal waveform which resembles Core Saturation.

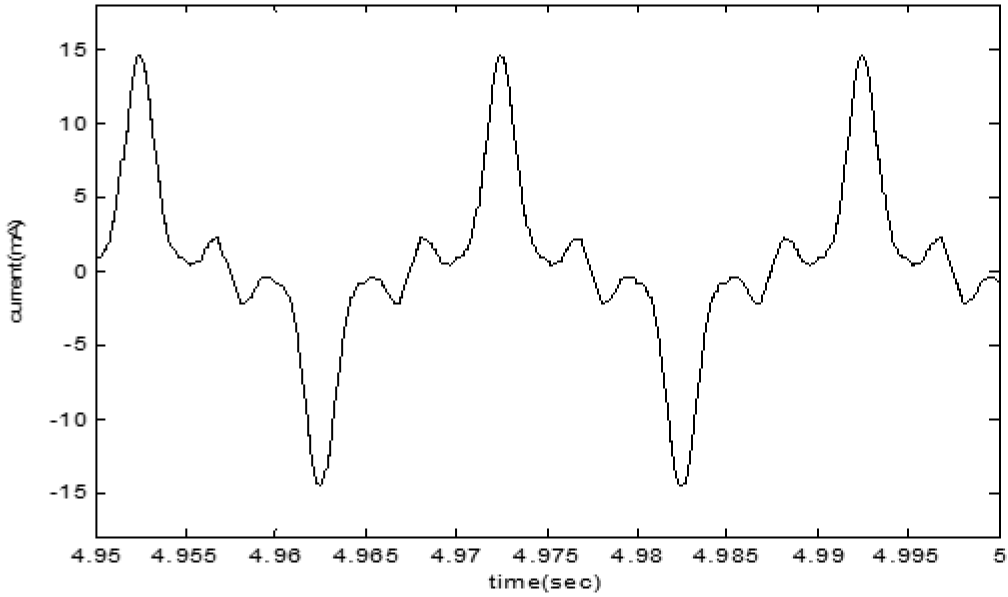

Fig.20. output current waveform

The FFT analysis shows that the THD of the Current waveform is 77.93% which is of a very high value. The main components for such THD is the 3rd harmonic and the 5th harmonic component. Apart from this there are also some percentage of 7th and 9th harmonics.

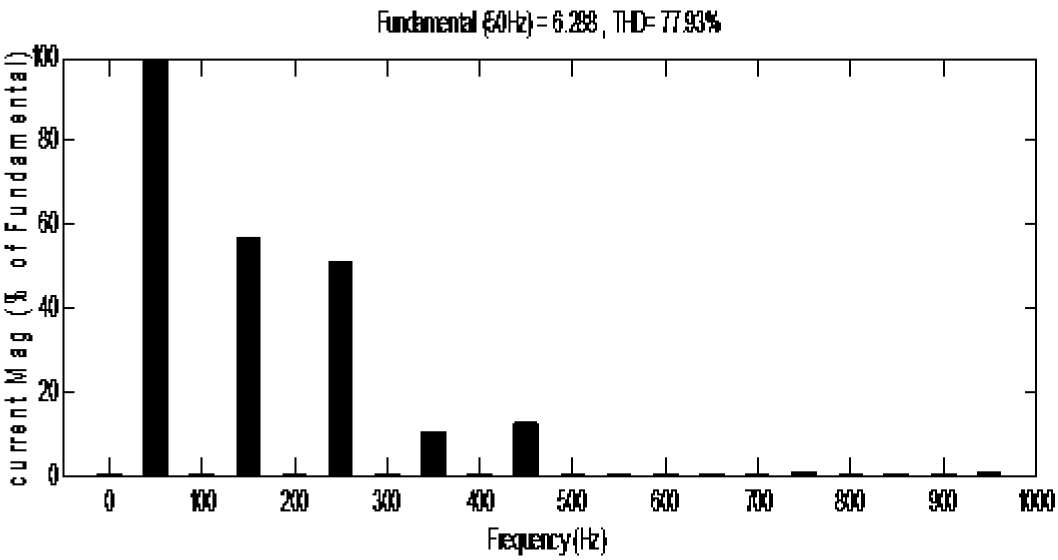

Fig 21. FFT analysis of current wave form

The flux waveform is also deviated from perfect sinusoid due to the Saturation characteristics incorporated in the Core with Non-sinusoidal supply Voltage. The peak Value of the flux waveform is 250mWb.

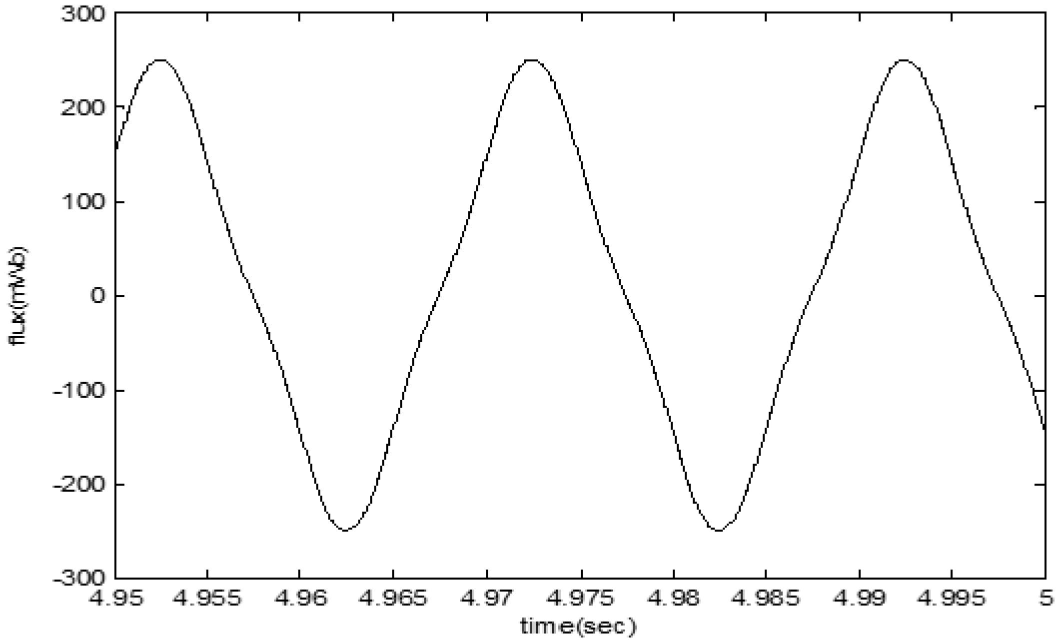

Fig.22. mutual flux waveform

The FFT analysis of the Flux waveform depicts the injection of 3rd harmonic component due to Non-sinusoidal supply Voltage and Core Saturation. The THD in the Flux is 11.05% as the fundamental component corresponds to 227mWb.

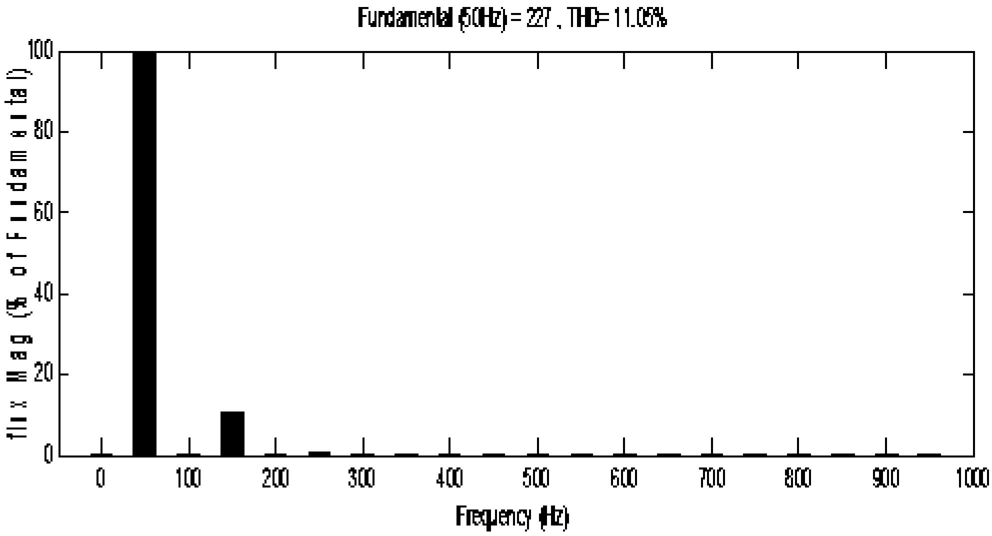

Fig.23. mutual flux waveform FFT analysis

4.3) Experimental Results of Open Circuit Test

The variation in excitation current was observed for different values of supply voltage.

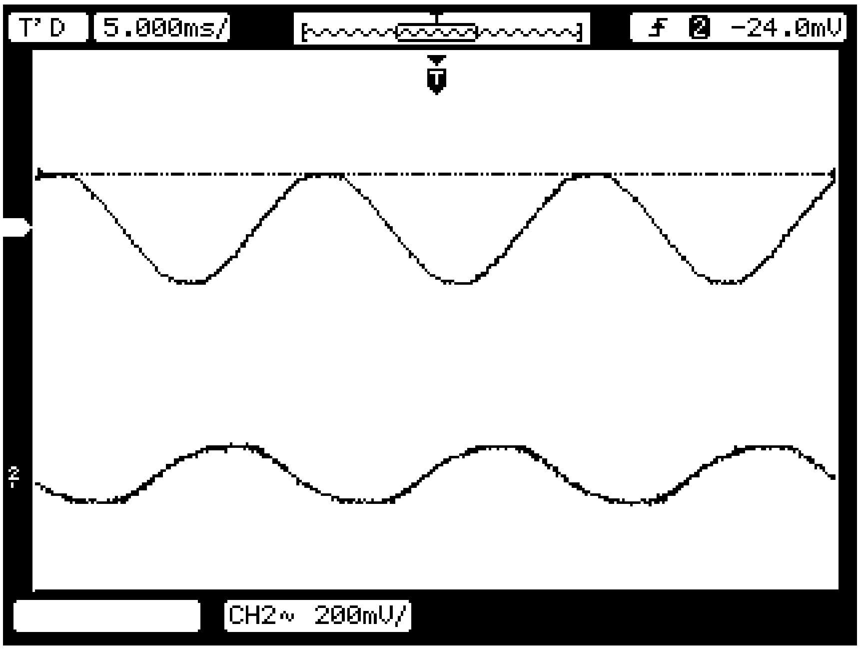

Fig.24. excitation current waveform for input sinusoidal voltage 40v.

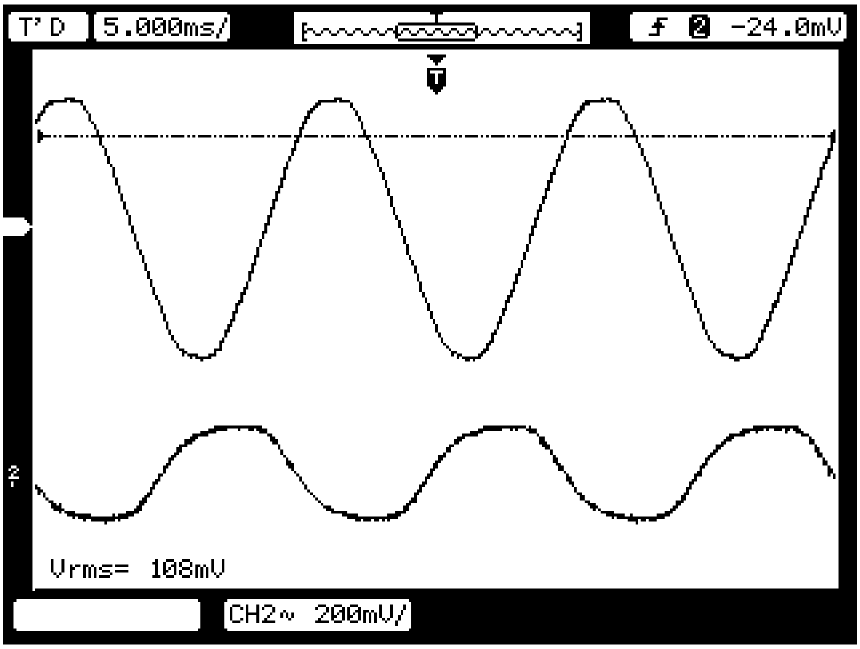

Fig.25. excitation current waveform for sinusoidal input voltage 70v

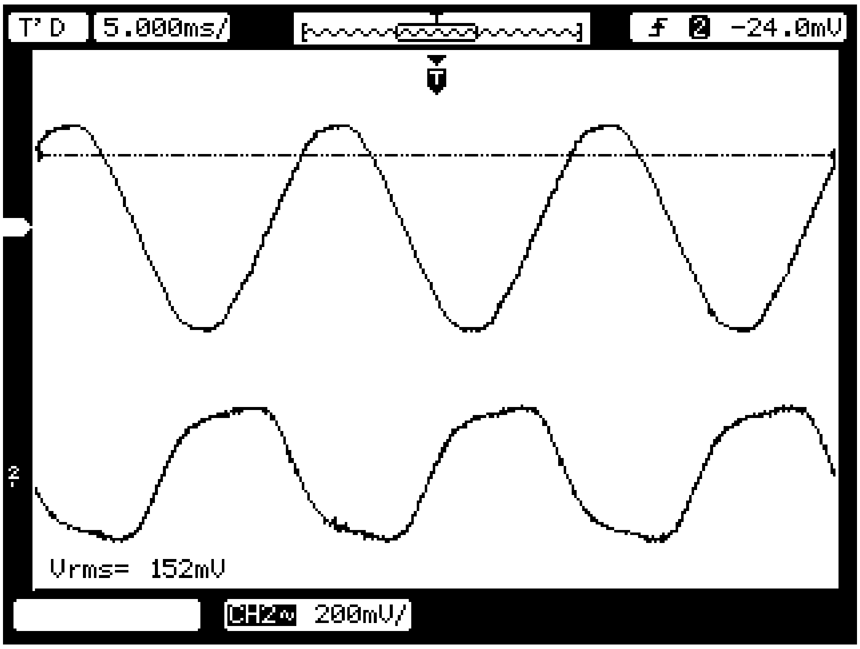

Fig.26.excitation current waveform for sinusoidal input voltage 110v

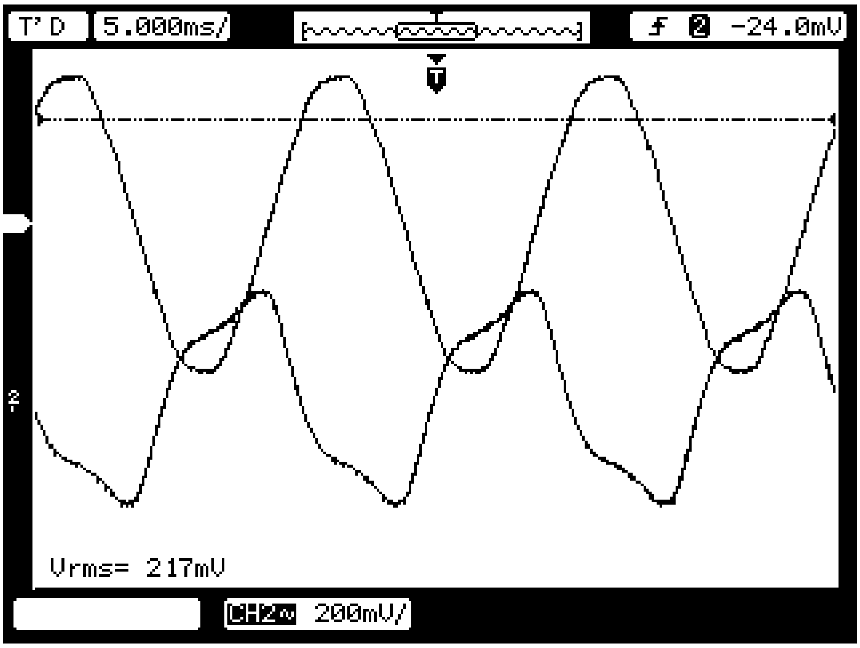

Fig.27. excitation current waveform for sinusoidal input voltage 160v

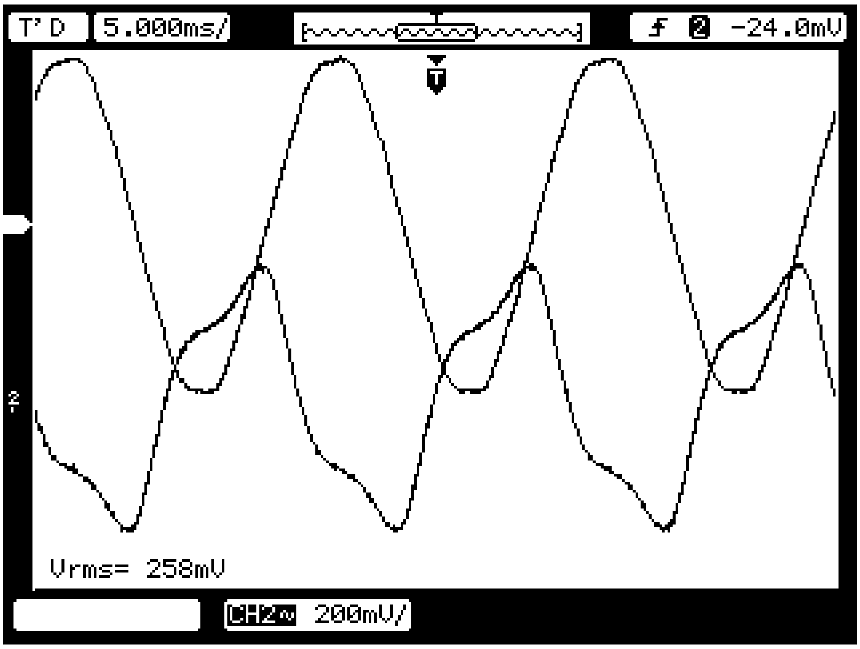

Fig.28. excitation current waveform for sinusoidal input voltage 200v

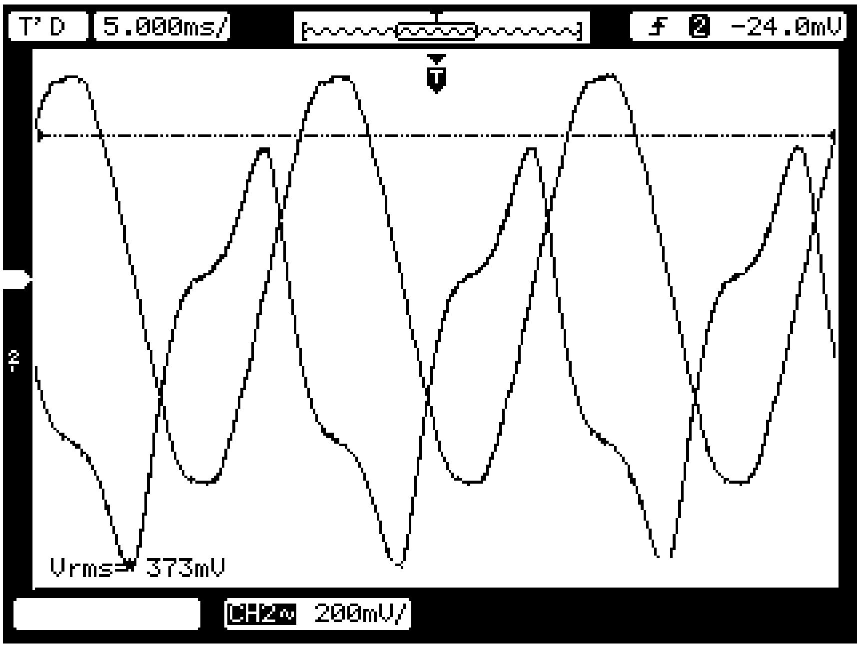

Fig.29. excitation current waveform for sinusoidal input voltage 220v

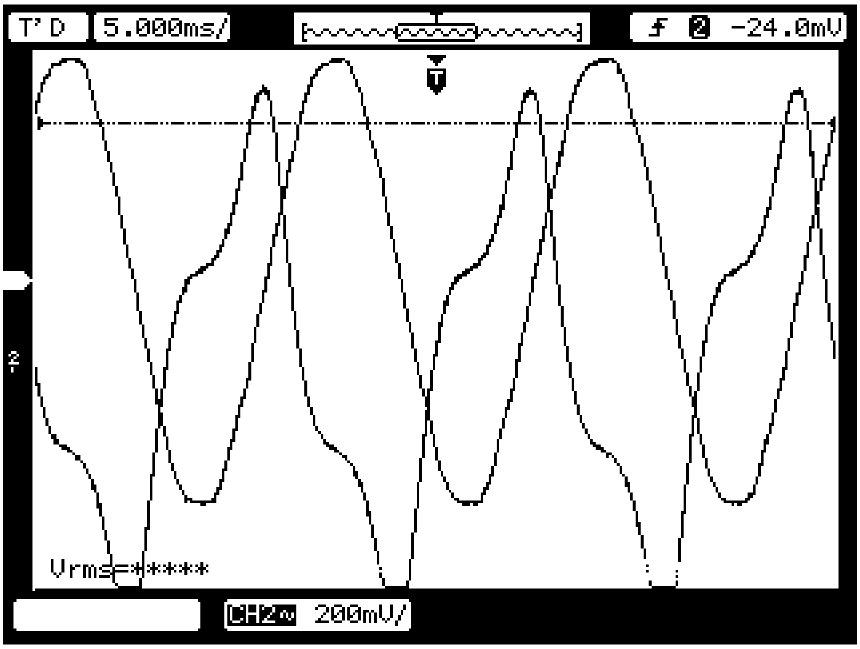

Fig.30. excitation current waveform for sinusoidal input voltage 240v

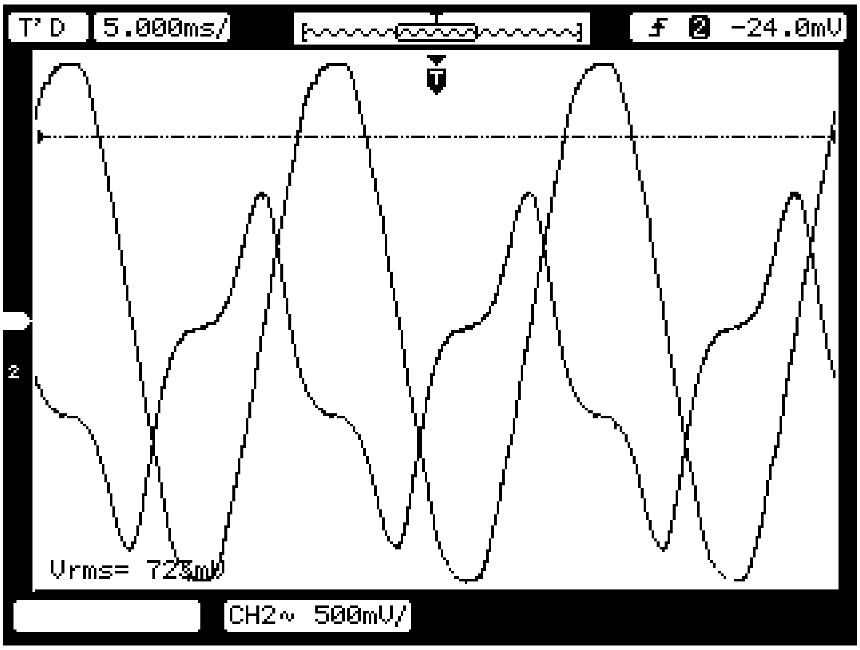

Fig.31. excitation current waveform for sinusoidal input voltage 280v

4.4) Simulations of Simulink Model of Transformer with Saturable Core:

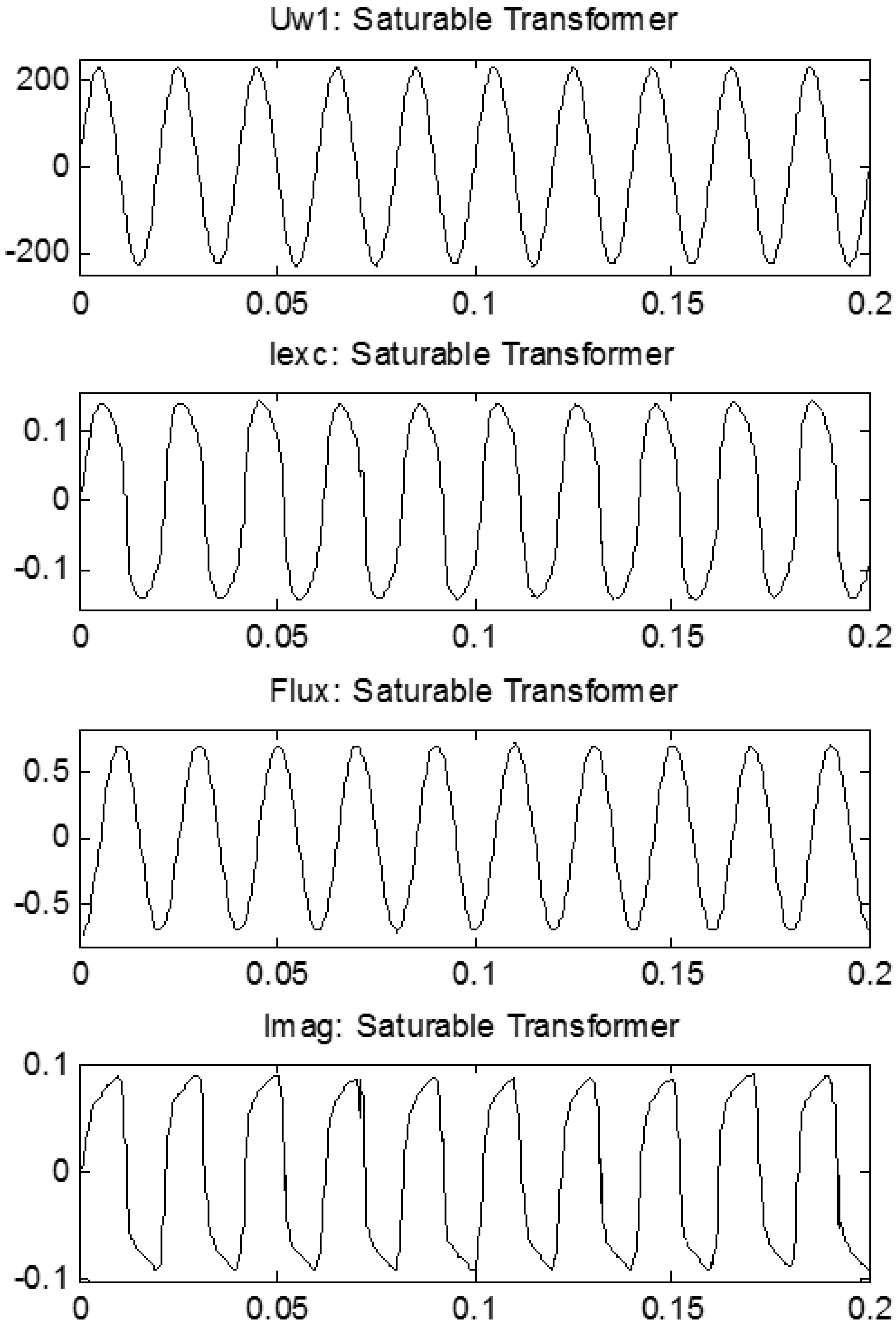

Fig.32. waveforms of sinusoidal input voltage to saturable transformer Simulink model.

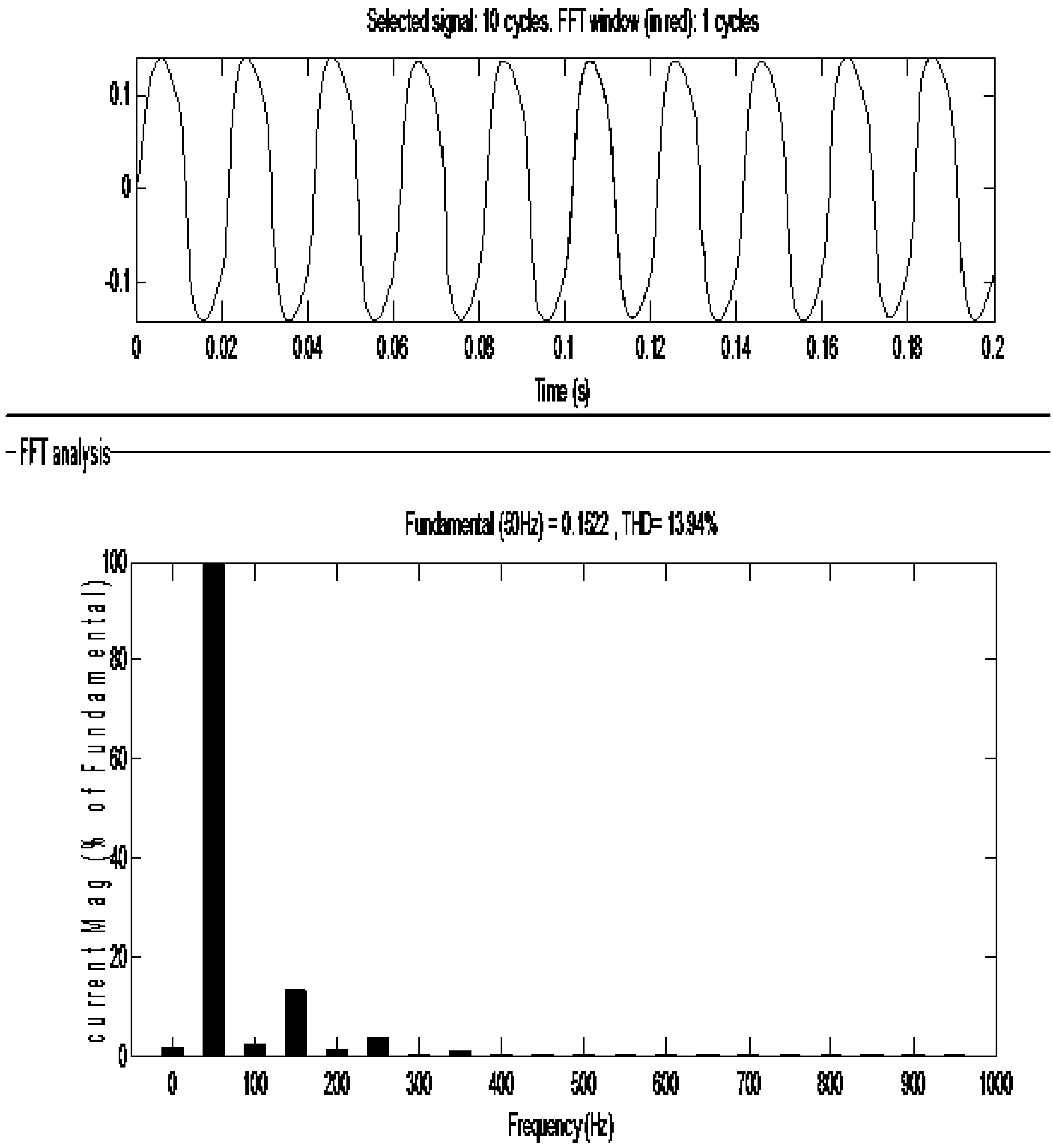

Fig.33. FFT analysis of excitation current waveform with sinusoidal input voltage

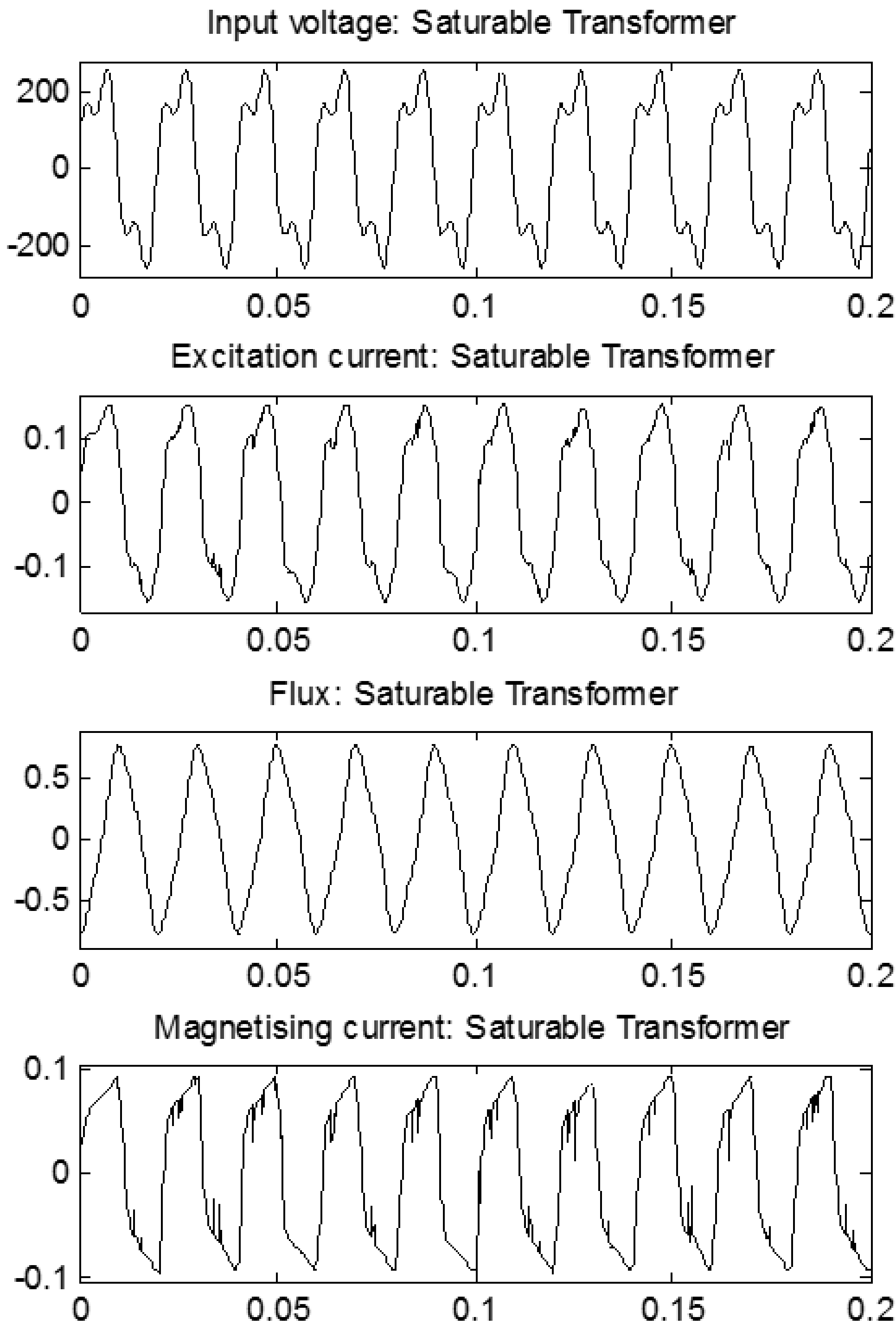

Fig.34. waveforms of non-sinusoidal input voltage to saturable transformer Simulink model.

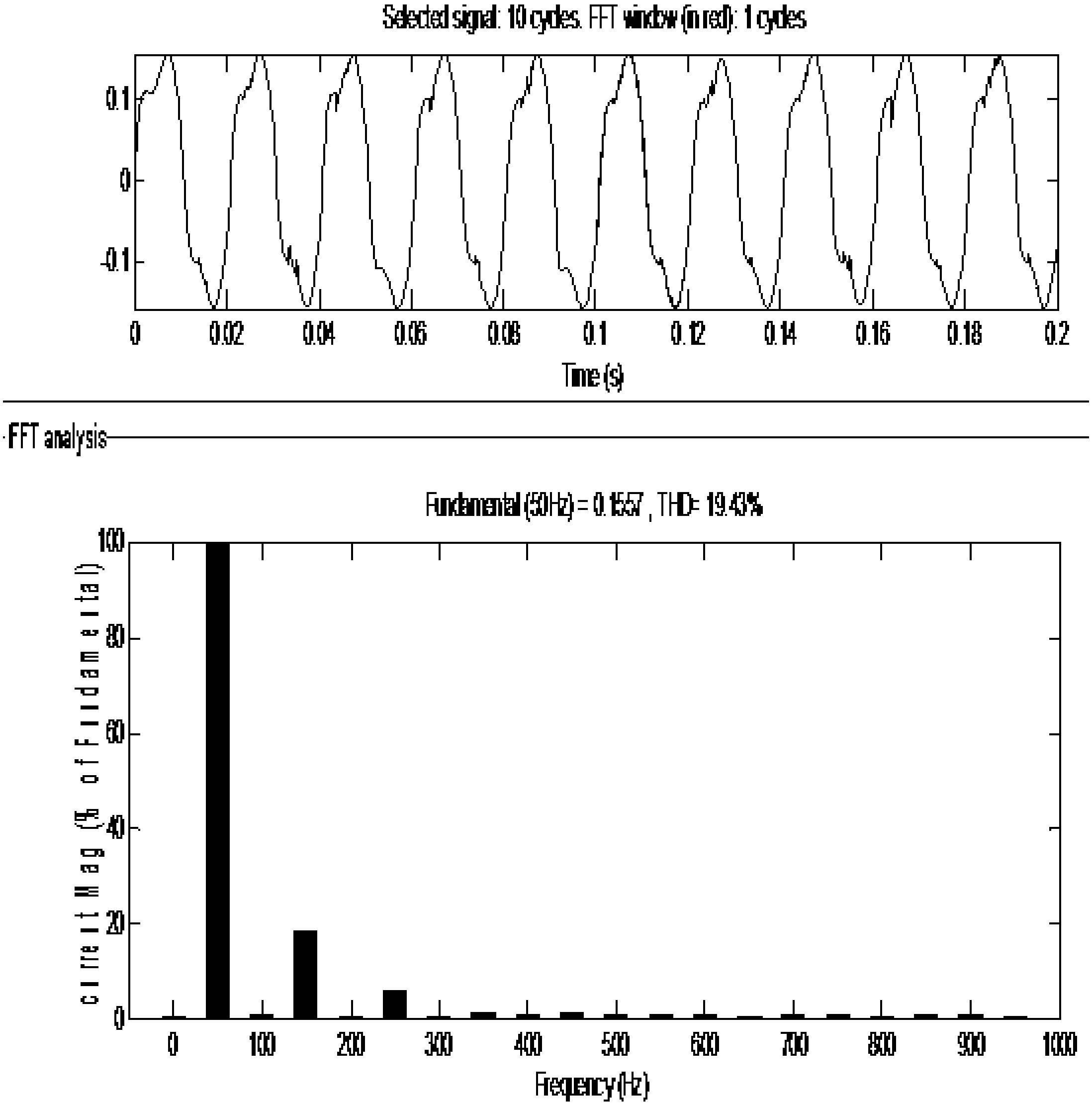

Fig.35. FFT analysis of excitation current waveform with non-sinusoidal input voltage

Chapter **5**

Conclusions and Future work

5.1 Conclusions

On applying the transformer with sinusoidal supply voltage the flux is sinusoidal while the excitation current waveform is sinusoidal or non-sinusoidal depending upon the region of operation. If the region of operation is below knee point then the current waveform follows the voltage waveform. If the region of operation is above knee point the saturation is automatically incorporated into the core which then distorts the excitation current. In case of non-sinusoidal supply voltage the flux gets saturated due to the saturation in the core leading to the excitation current with more total harmonic distortion .The eddy current losses increases quadratically as the RMS value of supply voltage increases. The modelling of the transformer is done without core saturation and with core saturation. And the results were observed. From the waveforms it is observed that with increase in non-linearity in the supply voltage the excitation waveform becomes more and more peaky thus increasing the third harmonic component. With the deviation of supply voltage from perfect sinusoid the flux also becomes non-sinusoidal. The total harmonic distortion increases with increase in saturation. for a sinusoidal input voltage of 230 v the THD of excitation current waveform of a saturable transformer was found to be 54.92 per cent whereas for a non-sinusoidal input with a magnitude of 230v fundamental frequency and 80v magnitude 3^{rd} harmonic component has a total harmonic distortion of 77.93 %.

5.2 Future work

As of now the three phase transformers are an integral part of our power system, modelling of three phase transformers are to be performed. The modelling of different connections of transformer delta-wye, wye-wye is to be performed. The autotransformer has to be simulated by using its winding its winding connections. The single phase transformer model has to be modelled with a non-linear load.

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